Bounds on Hop Distance in Greedy Routing Approach in Wireless Ad Hoc Networks

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Abstract. Wireless ad hoc networks are characterized by random location of nodes, and the routes are generally multihop. A quantitative understanding of the relation between source-to-destination Euclidean distance and the hop count could provide us the knowledge about important network parameters such as granularity of localization, traffic load, end-to-end delay, delay jitter, and transmit power consumption. In this paper, we present an analytic approach to evaluate the average and bounds on hop count for a given source-to-destination Euclidean distance in greedy routing approach in wireless ad hoc networks. We also show that from the distribution characteristic of per-hop progress, the bounds on Euclidean distance from a given hop count can be obtained numerically. Our analytic and numerical results are verified by discrete event network simulations.

Categories and subject descriptors: C.2 [**Computer Communication Networks**].

1 Introduction

The need for efficient and flexible network access has led to the recent proliferation of research on multihop wireless networks [1]. Due to the ad hoc nature of network set up, in most cases the nodes are randomly located with respect to each other. Lack of any predetermined infrastructure in such a network necessitates that any two neighbor nodes are connected via wireless communication channel, and a message has to go through several intermediate 'peer' nodes to reach its desired destination. Such an ad hoc multihop wireless network could be mobile or relatively static, but the random nature of node distribution prevails.

There have been numerous works on various aspects of ad hoc multihop wireless networks in the literature, including extensive studies on ad hoc routing protocols [2],[3]. Geographic location aware greedy routing approach, which attempts to forward the data to a node closest to the destination, is one of the widely studied protocols [4],[5],[6]. Although greedy 'shortest path' packet forwarding does not guarantee minimum energy or high capacity routes [7],[8], its importance lies in its simplicity and scalability. There also have been numerous variants to the greedy forwarding approach that attempt to circumvent the routing voids in different ways [4],[5],[9]. In this paper, however, we will focus on simple greedy routing approach.

We are interested in correlating the Euclidean distance between two nodes and the number of hops required to connect them in a greedy routing approach in an ad hoc multihop scenario. There could be several interesting areas where the knowledge of the relationship between Euclidean distance and hop count could be useful: (a) In a routing approach, given an Euclidean distance, the knowledge on the distribution characteristic of hop count can determine the key network aspects such as data and control traffic load, packet delay and delivery rate [10]. (b) In a virtual coordinate based positioning and routing approach (e.g., [11],[12],[13]), approximate relative distance can be determined from the hop count information. It is also possible to derive an approximate locations of other nodes by combining hop distances from nodes whose locations are known. (c) Also, the average and higher order moments of progress per hop enables us to capture the parameters like transmit power consumption and path loss.

The randomness of node location implies that for a given source-to-destination Euclidean distance, the number of hops is non-deterministic, and similarly, for a given number of hops, the Euclidean distance between a source and a destination is non-deterministic. It is intuitive that given a source-to-destination distance and a network density, the number of hops could be within a certain range. A few recent papers (e.g., [11],[13],[14]) addressed the problem of establishing the relationship between Euclidean distance and hop distance. However, the evaluation of tight bounds relating these distances is still missing.

In this paper, we present an analytic approach to obtain the bounds on number of hops for a given source-to-destination Euclidean distance. We consider a greedy routing approach that attempts to minimize the remaining distance to the destination in each hop, we call this approach *least remaining distance* (LRD) forwarding. In determining the bounds on hop count from Euclidean distance, or vice versa, we do not consider physical layer channel condition based link availability. Rather, if a node is within the transmission range of another node, they are considered reachable to each other. In other words, although we use the term "wireless" multihop networks – because it is only the natural way to connect the nodes in an ad hoc network, in this work we are not concerned about distance and channel interference dependent link quality/availability.

Our main contributions in this paper are the following. (*i*) We derive the distribution characteristics of the remaining Euclidean distance and the forward progress in one hop, and show that the average progress per hop is a function of node density and the current distance to the destination. We show that the average one-hop progress based on the initial source-to-destination distance can give us a good approximation of average hop count. (*ii*) The bounds on the hop distance are evaluated numerically from the distribution characteristic of the remaining distance in one hop. (*iii*) We also show that from the characteristic of forward progress per hop it is possible to numerically compute the bounds on Euclidean distance from a given hop count. (*iv*) We outline a few potential network parameter estimations where the knowledge of the relationship between hop count and Euclidean distance can be very useful.

The rest of this paper is organized as follows. We briefly review the past related work in Section 2. In Section 3, we present our analytic approach to obtain the bounds on hop distance. The analytic approximation error is also evaluated in this section. Section 4 contains the analytic, numerical, and simulation results. In Section 5, we discuss a few potential applications of the relationship between hop count and Euclidean distance. We conclude the paper in Section 6.

2 Related Work

Significant research have been carried out on the development and optimization of ad hoc routing protocols from energy efficiency and network capacity view point (see e.g., [2],[3]). Mathematical modeling of ad hoc networks has also been looked into by several researchers. We briefly survey the works related to our present studies.

In field sensor networks, the approximate position information is useful in devising simple routing approaches and for correlating the collected (sensed) data with the geographic location. This helps the monitoring and control station to take appropriate action at the desired location [15]. For simplifying the problem of localization and routing using low-cost sensor nodes that are equipped with only basic sensing and routing functionalities, there have been recent proposals on hop count metric based virtual positioning of nodes [12],[13], that attempt to get away with the global positioning system (GPS) based and other geographic localization approaches. This hop count based virtual positioning and routing approach works on the basic premise that given a hop count between two nodes, a bound on Euclidean distance between them can be found.

In connection to maximizing one-hop transmission capacity, Kleinrock and Silvester [16] and Takagi and Kleinrock [17] evaluated the average of maximum progress towards the destination in one hop, called most forward within radius R (MFR), where backward movement of a packet is allowed in case no better forwarding node is available. Later, Hou and Li [18] studied MFR and a variant, called most forward with variable radius (MVR), without allowing backward progress. Again, the interest was evaluating the one-hop capacity and the influence of interference associated with different transmission range properties. In context of road networking, Mathar and Mattfeldt [19] studied the optimal transmission range problem in one dimension and obtained the expected per-hop progress. Hekmat and Mieghen [14] studied the expected hop count as a function of channel shadowing in wireless ad hoc networks. For the regular two dimensional lattice networks, an expression for obtaining the expected hop count was given. For a geometric random graph, such as an ad hoc multihop wireless network, the evaluation was simulation based. He *et al.* [20] applied greedy routing and conducted simulation based performance studies where packet forwarding essentially minimizes the remaining distance for minimizing the end-to-end packet delay. Bischoff and Wattenhofer [11] studied positioning of field sensor nodes from the anchor nodes' location information and based on the hop counts from multiple anchors. They have proved that one dimensional distance can be bounded by hop count, but the exact bound was not shown.

In the following section we will analyze the distribution characteristics of the remaining distance and the distance progress in one hop in greedy routing that minimizes the remaining distance towards the destination.

3 Analysis on Bounds

We consider uniformly randomly placed nodes in a given location space. All nodes have equal, omnidirectional transmission pattern of range R. Since there is a direct dependence of geometric distance between nodes on the connectivity, the network is modeled as an undirected geometric random graph [21]. The greedy forwarding approach, which we call least remaining distance (LRD) forwarding, attempts to minimize the remaining distance in each hop. LRD forwarding is different from the maximum forwarding with fixed radius (MFR) approach that was studied in [16],[17],[18], and the maximum forwarding with variable radius (MVR) approach that was studied in [18]. In particular, as observed in [17], although MFR and MVR are greedy approaches and ensure the maximum progress in the direction of the destination, they do not guarantee minimizing the remaining distance to the destination. In contrast, our LRD approach minimizes the remaining distance. The LRD forwarding captures the case for the nodes with fixed transmission radius as well as the case with variable radius, i.e., without and with transmission power control.

As in [18], we try to avoid backward movement if no forwarding node closer to the destination is found. In case of no available forwarding nodes, the data packet to be forwarded is considered to be dropped (lost). For simplicity of the analysis, we consider a node to be a potential forwarder if it is in the half circle of the transmission range of a node towards the destination (the entire shaded region in Fig. 1). Precisely, this approach does not guarantee that the remaining distance would be always lesser than the current distance. As shown in Fig. 1, in our analysis a neighbor of node S located anywhere in the total shaded region can be a potential forwarder. But, if the selected forwarding node is located in the densely shaded region, the remaining distance to the destination D would be larger than the current distance between S and D. However, as we will see in Section 4, with high node density such possibility of potential "backward movement" is very insignificant – even when the densely shaded region is included.

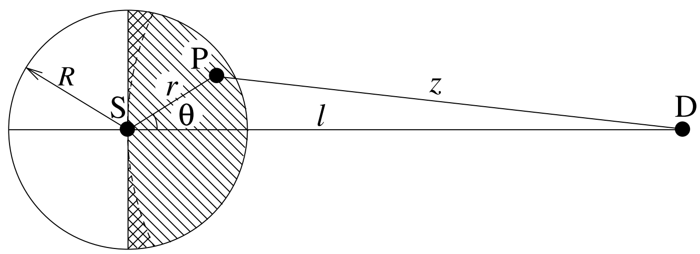

Figure 1: Pictorial representation of greedy forwarding considered in our analysis. P is a potential forwarder of a node S to the destination D. P could be located anywhere inside the shaded region.

We denote the distance between a node S and the destination D by l. Referring to Fig. 1, let P be a potential forwarder of S, randomly located at a distance r and angle θ, and let the remaining distance from P to D be z. Our immediate goal is to find the characteristics of z.

In the following, unless otherwise stated, a symbol in bold denotes a random variable (RV) and a symbol in italics denotes a realization (sample value).

3.1 Least remaining distance

With respect to the node S, the random position of P is characterized by the following probability density functions (pdf's) of $\boldsymbol{r}$ and $\boldsymbol{\theta}$:

$$f_{\boldsymbol{r}}(r) = \begin{cases} \frac{2r}{R^2}, & 0 \le r \le R \\ 0, & \text{elsewhere} \end{cases}$$

and

$$f_{\boldsymbol{\theta}}(\theta) = \begin{cases} \frac{1}{\pi}, & -\frac{\pi}{2} \le \theta \le \frac{\pi}{2} \\ 0, & \text{elsewhere.} \end{cases}$$

Since $\boldsymbol{r}$ and $\boldsymbol{\theta}$ and mutually independent, their joint pdf is given by

$$f_{\boldsymbol{r}\boldsymbol{\theta}}(r,\theta) = \begin{cases} \frac{2r}{\pi R^2}, & 0 \le r \le R \text{ and } -\frac{\pi}{2} \le \theta \le \frac{\pi}{2} \\ 0, & \text{elsewhere.} \end{cases} \tag{3.1}$$

Let $\boldsymbol{x} = \boldsymbol{r}\cos\boldsymbol{\theta}$ and $\boldsymbol{y} = \boldsymbol{r}\sin\boldsymbol{\theta}$. From (3.1) the joint pdf of the transformed variables $\boldsymbol{x}$ and $\boldsymbol{y}$ is obtained as

$$f_{\boldsymbol{x}\boldsymbol{y}}(x,y) = \begin{cases} \frac{2}{\pi R^2}, & 0 \le x \le R, \; -R \le y \le R, \text{ and } x^2+y^2 \le R^2 \\ 0, & \text{elsewhere.} \end{cases} \tag{3.2}$$

From geometry, we have $\boldsymbol{z} = \sqrt{(l-\boldsymbol{x})^2 + \boldsymbol{y}^2}$. Introducing an auxiliary variable $\boldsymbol{w} = \boldsymbol{x}$, from (3.2) the joint pdf of $\boldsymbol{z}$ and $\boldsymbol{w}$ is obtained as

$$f_{\boldsymbol{z}\boldsymbol{w}}(z,w) = \begin{cases} \frac{4z}{\pi R^2\sqrt{z^2-(l-w)^2}}, & \begin{array}{l} l-R \le z \le l \text{ and } l-z \le w \le \frac{l^2+R^2-z^2}{2l} \\ \text{or,} \\ l \le z \le \sqrt{l^2+R^2} \text{ and } 0 \le w \le \frac{l^2+R^2-z^2}{2l} \end{array} \\ 0, & \text{elsewhere.} \end{cases} \tag{3.3}$$

Integrating (3.3) over w, we have the pdf of $\boldsymbol{z}$ as

$$f_{\boldsymbol{z}}(z) = \begin{cases} \frac{4z}{\pi R^2}\left[\frac{\pi}{2} - \arcsin\left(\frac{l^2-R^2+z^2}{2lz}\right)\right], & l-R \le z \le l \\ \frac{4z}{\pi R^2}\left[\arcsin\left(\frac{l}{z}\right) - \arcsin\left(\frac{l^2-R^2+z^2}{2lz}\right)\right], & l \le z \le \sqrt{l^2+R^2} \\ 0, & \text{elsewhere.} \end{cases} \tag{3.4}$$

The corresponding cumulative distribution function (cdf) is $F_{\boldsymbol{z}}(z) = \int_{-\infty}^{z} f_{\boldsymbol{z}}(t)dt$. From (3.4), the remaining distance from an arbitrarily located forwarding node P to the destination D can be known probabilistically.

To find the least remaining distance (LRD) to the destination D, we have to first obtain the number of potential forwarders, i.e., the number of nodes in the shaded region in Fig. 1. Let this number be n. For a uniformly random node distribution with a given density $\rho = \frac{N}{A}$, where N is the total number of nodes distributed over a location space of area A, using Poisson approximation we have $n = \frac{\rho\pi R^2}{2}$. We denote the remaining distances from n potential forwarding nodes as $\boldsymbol{z_1}, \; \boldsymbol{z_2}, \; \cdots, \boldsymbol{z_n}$. To determine the forwarding node corresponding to the LRD to D, we have to obtain

$$\boldsymbol{\delta} = \min\{\boldsymbol{z_1}, \; \boldsymbol{z_2}, \; \cdots, \; \boldsymbol{z_n}\}.$$

Since the nodes are uniformly random distributed, $\boldsymbol{z_i}\ \forall\ i = 1$ to n are independent and identically distributed RVs, Therefore, following [22, Chapter 6], the pdf of $\boldsymbol{\delta}$ is given by

$$f_{\boldsymbol{\delta}}(\cdot) = n f_{\boldsymbol{z}}(\cdot) R_{\boldsymbol{z}}^{n-1}(\cdot) \tag{3.5}$$

where $f_{\boldsymbol{z}}(z)$ is given by (3.4), and $R_{\boldsymbol{z}} \triangleq F_{\boldsymbol{z}}^c = 1 - F_{\boldsymbol{z}}$ is the complementary cdf of $\boldsymbol{z}$. The expression for $R_{\boldsymbol{z}}(z)$ is

$$R_{\boldsymbol{z}}(z) = \begin{cases} 1, & z < l-R \\ -\frac{2}{\pi R^2}\left[\frac{\pi z^2}{2} - z^2\arcsin\left(\frac{z^2+l^2-R^2}{2lz}\right) - \frac{1}{2}\sqrt{4R^2l^2-(z^2-l^2-R^2)^2} \right. \\ \left. \quad + R^2\arcsin\left(\frac{z^2-l^2-R^2}{2lR}\right)\right], & l-R \le z \le l \\ -\frac{2}{\pi R^2}\left[z^2\arcsin\left(\frac{l}{z}\right) - z^2\arcsin\left(\frac{z^2+l^2-R^2}{2lz}\right) - \frac{1}{2}\sqrt{4R^2l^2-(z^2-l^2-R^2)^2} \right. \\ \left. \quad + R^2\arcsin\left(\frac{z^2-l^2-R^2}{2lR}\right) + l\sqrt{z^2-l^2}\right], & l \le z \le \sqrt{l^2+R^2} \\ 0, & z > \sqrt{l^2+R^2} \end{cases} \tag{3.6}$$

Substituting (3.4) and (3.6) in (3.5), we find the pdf of the least remaining distance $\boldsymbol{\delta}$ as

$$
f_{\boldsymbol{\delta}}(\delta) = \begin{cases} n\left(\frac{2}{\pi R^2}\right)^n 2\delta \left[\frac{\pi}{2} - \arcsin\left(\frac{\delta^2+l^2-R^2}{2l\delta}\right)\right] \left[\delta^2 \arcsin\left(\frac{\delta^2+l^2-R^2}{2l\delta}\right) \right. \\ \quad \left. +\frac{1}{2}\sqrt{4R^2l^2 - (\delta^2 - l^2 - R^2)^2} - R^2 \arcsin\left(\frac{\delta^2-l^2-R^2}{2lR}\right) - \frac{\pi\delta^2}{2}\right]^{n-1}, & l - R \leq \delta \leq l \\ n\left(\frac{2}{\pi R^2}\right)^n 2\delta \left[\arcsin\left(\frac{l}{\delta}\right) - \arcsin\left(\frac{\delta^2+l^2-R^2}{2l\delta}\right)\right] \left[\delta^2 \arcsin\left(\frac{\delta^2+l^2-R^2}{2l\delta}\right) \right. \\ \quad +\frac{1}{2}\sqrt{4R^2l^2 - (\delta^2 - l^2 - R^2)^2} \\ \quad \left. -R^2 \arcsin\left(\frac{\delta^2-l^2-R^2}{2lR}\right) - \delta^2 \arcsin\left(\frac{l}{\delta}\right) - l\sqrt{\delta^2 - l^2}\right]^{n-1}, & l \leq \delta \leq \sqrt{l^2+R^2} \\ 0, & \text{elsewhere.} \end{cases} \tag{3.7}
$$

3.2 Average progress in one hop

Once the characteristic of least remaining distance in one hop $\boldsymbol{\delta}$ is known, the maximum forward progress $\boldsymbol{\varepsilon}$ can be simply obtained from the relation $\boldsymbol{\varepsilon} = l - \boldsymbol{\delta}$. Correspondingly the pdf of progress in one hop towards the destination $\boldsymbol{\varepsilon}$ is given by

$$
f_{\boldsymbol{\varepsilon}}(\varepsilon) = f_{\boldsymbol{\delta}}(l - \varepsilon).
$$

Simple substitution of $f_{\boldsymbol{\delta}}(\cdot)$ from (3.7) gives

$$
f_{\boldsymbol{\varepsilon}}(\varepsilon) = \begin{cases} n\left(\frac{2}{\pi R^2}\right)^n 2(l-\varepsilon) \left[\frac{\pi}{2} - \arcsin\left(1 + \frac{\varepsilon^2-R^2}{2l(l-\varepsilon)}\right)\right] \left[(l-\varepsilon)^2 \arcsin\left(1 + \frac{\varepsilon^2-R^2}{2l(l-\varepsilon)}\right) \right. \\ \quad \left. +\frac{1}{2}\sqrt{4R^2l^2 - (\varepsilon^2 - R^2 - 2l\varepsilon)^2} - R^2 \arcsin\left(\frac{\varepsilon^2-R^2-2l\varepsilon}{2lR}\right) - \frac{\pi(l-\varepsilon)^2}{2}\right]^{n-1}, & R \geq \varepsilon \geq 0 \\ n\left(\frac{2}{\pi R^2}\right)^n 2(l-\varepsilon) \left[\arcsin\left(\frac{l}{l-\varepsilon}\right) - \arcsin\left(1 + \frac{\varepsilon^2-R^2}{2l(l-\varepsilon)}\right)\right] \times \\ \quad \left[(l-\varepsilon)^2 \arcsin\left(1 + \frac{\varepsilon^2-R^2}{2l(l-\varepsilon)}\right) + \frac{1}{2}\sqrt{4R^2l^2 - (\varepsilon^2 - R^2 - 2l\varepsilon)^2} \right. \\ \quad \left. -R^2 \arcsin\left(\frac{\varepsilon^2-R^2-2l\varepsilon}{2lR}\right) - (l-\varepsilon)^2 \arcsin\left(\frac{l}{l-\varepsilon}\right) - l\sqrt{\varepsilon^2 - 2l\varepsilon}\right]^{n-1}, & 0 \geq \varepsilon \geq l - \sqrt{l^2+R^2} \\ 0, & \text{elsewhere.} \end{cases} \tag{3.8}
$$

From (3.8), the average progress per hop towards the destination $\overline{\varepsilon}$ is obtained using

$$
\overline{\varepsilon} = \int_{l-\sqrt{l^2+R^2}}^{R} t f_{\boldsymbol{\varepsilon}}(t) dt. \tag{3.9}
$$

The distribution of $\boldsymbol{\delta}$ in (3.7) (and hence that of $\boldsymbol{\varepsilon}$ in (3.8)) would give us the probabilistic bounds of hop count to the destination.

Before presenting the numerical results on hop bounds in LRD forwarding, we evaluate the error in approximating the forwarding region.

3.3 Effect of approximating the area of greedy forwarding

As we have stated in the beginning of Section 3, referring to Fig. 1, for simplicity of analysis we included the densely shaded region, although a selected forwarding node from that region actually increases the remaining distance to the destination. Below we will show, given a node density, what is the chance of increased remaining distance when the nodes in the densely shaded region are also considered. The same approach will also show the effect of node density on successful greedy forwarding.

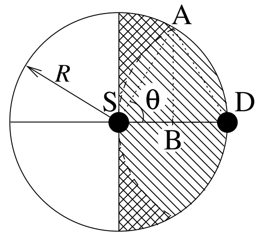

Figure 2: Probability of error in greedy forwarding when the entire shaded region is considered.

It is intuitively clear that as the destination node is approached, the area of densely shaded region and hence the chance of increased remaining distance increases. Fig. 2 shows the extreme case when S and D are marginally beyond each other's range that gives us the maximum chance of increased remaining distance.

Poisson approximation of node distribution gives us

$$\begin{aligned} \Pr[\text{at least } \textit{one} \text{ neighbor in an area } a] &= \sum_{k=1}^{N-1} \frac{(\rho a)^k}{k!} e^{-\rho a} \\ &\approx 1 - e^{-\rho a} \end{aligned}$$

where $\rho = \frac{N}{A}$ is the node density in the network, with $N \gg 1$ and $a \ll A$. Thus, referring to Fig. 2, if only the greedy forwarding zone, i.e., the lightly shaded region with area a_s were considered, the probability of successful greedy forwarding P_s would have been

$$P_s = 1 - e^{-\rho a_s} \tag{3.10}$$

where, from simple geometry we have $a_s = 2R^2 \left(\frac{\pi}{3} - \frac{\sqrt{3}}{4} \right)$. When entire shaded region of area a_{s_+} is considered, the probability of successful forwarding P_{s_+} becomes

$$P_{s_+} = 1 - e^{-\rho a_{s_+}} \tag{3.11}$$

where $a_{s_+} = \frac{\pi R^2}{2}$.

Note that, given a node density, P_s in (3.10) is the least possible success probability, as the area a_s is the minimum when S and D are just R distance apart. Correspondingly, the probability of error in successful greedy forwarding P_e, i.e., the chance of increased remaining distance when the densely shaded region is included, is

$$\begin{aligned} P_e &\leq P_{s_+} - P_s \\ &\leq e^{-\rho a_s} - e^{-\rho a_{s_+}}. \end{aligned} \tag{3.12}$$

Using (3.10), (3.11), and (3.12), for a given node density, the chance of increased remaining distance due to approximating the area of greedy forwarding can be obtained. Also, the effect of node density on minimizing the chance of increased remaining distance P_e can be found from (3.10). In other words, to minimize P_e, we can determine from (3.10) how much node density we need such that P_s approaches to one.

Numerical results on analytic approximation error and the effect of node density on successful greedy forwarding will be shown in the next section.

4 Results and Discussion

In this section, we present the results from analysis, and wherever necessary, we verify the analysis with network simulations. In the simulation we consider a $400m \times 400m$ location space, where the nodes are uniformly random distributed. The transmission range of a node is fixed at $R = 10m$. The total number of nodes is varied appropriately to attain a desired node density $\rho = \frac{2n}{\pi R^2}$, i.e., to have the same average number of potential forwarding neighbors of a node n as in the analysis. To minimize the "border effect" [23], the source and destination nodes are considered twice the range inside the boundary.

4.1 Average number of hops

We first study the nature of the remaining distance in LRD forwarding. Fig. 3 shows the probabilistic variation of the remaining distance $\boldsymbol{\delta}$ in a single hop. The corresponding pdf of

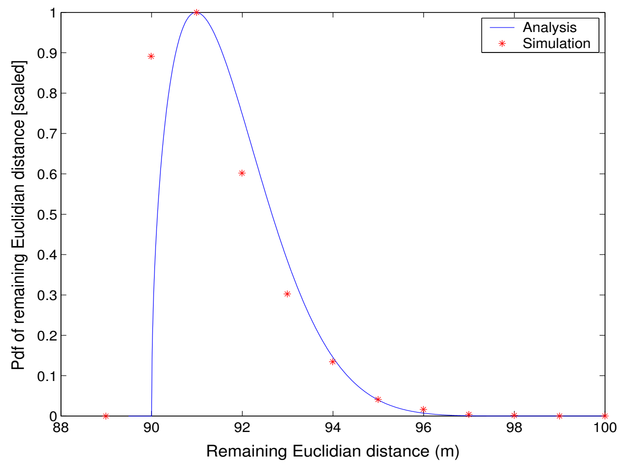

Figure 3: Pdf of remaining distance in one hop. $n = 10$, $l = 100m$.

distance progress in one hop $\boldsymbol{\varepsilon}$ will have the nature complementary to that of $\boldsymbol{\delta}$.

Using (3.7) and (3.8), in Table 1 we show the average progress in Euclidean metric towards the destination for different source-to-destination distance l and different node densities ρ (or, average number of forwarding neighbors n). The data in Table 1 indicate that average progress

Table 1: Average distance covered in one hop, $\overline{\varepsilon}$ (in m).

l (in m)	$n = 5$	$n = 10$	$n = 15$
30	7.164	8.128	8.548
100	7.366	8.275	8.667
400	7.422	8.315	8.699

is slowly varying with respect to l and a function of ρ and R. We study the average number of hops recursively as well as via a simple approximate approach, using (3.7) and (3.8). In the numerical recursion (Monte-Carlo simulation) approach, the least remaining distance δ obtained from (3.7) at each hop is considered as the current distance l to the destination in the next hop, and the process is repeated until the destination is reached. The approximate approach is based on the observation in Table 1. Considering the slowly varying nature of the average progress $\overline{\varepsilon}$ with respect to the source-to-destination distance l, we evaluate $\overline{\varepsilon}$ only once using (3.9) and (3.8), where we substitute l with $\frac{l}{2}$. To cover a distance l, the approximate average number of hops $\overline{h}$ is obtained as $\overline{h} = \frac{l}{\overline{\varepsilon}}$. Table 2 shows that the approximate hop

Table 2: Average number of hops $\overline{h}$ required to cover a distance.

l	$n = 5$		$n = 10$		$n = 15$	
(in m)	Recursion	Approximate	Recursion	Approximate	Recursion	Approximate
30	4.37	4.434	4.042	3.839	4.006	3.622
100	13.947	13.727	12.496	12.17	12.0	11.601
400	54.504	54.031	48.684	48.185	46.559	46.042

count matches well with the recursively obtained data. This implies that for a given source-to-destination Euclidean distance one can have an estimate of number of hops from the average progress in only one hop, obtained with a suitably set distance to the destination, which is much simpler and less time consuming than the recursive approach. As we will see in Section 4.2, these results are also corroborated via network simulation.

4.2 Bounds on number of hops

For obtaining the upper and lower limits on number of hops for a given source-to-destination distance, we conduct numerical (Monte-Carlo) simulations using (3.8) and verify the results via network simulations. Figs. 4 and 5 show the probability mass function (pmf) of number of hops for different node densities and source-to-destination distances. It is observed that in general, compared to the upper limit, the lower range of number of hops is characterized by sharper decay of discrete probabilities. This is because, for any node distribution and density there is a deterministic lowest possible lower limit $\lceil \frac{l}{R} \rceil$, whereas because of random distribution there is no such deterministic upper limit.

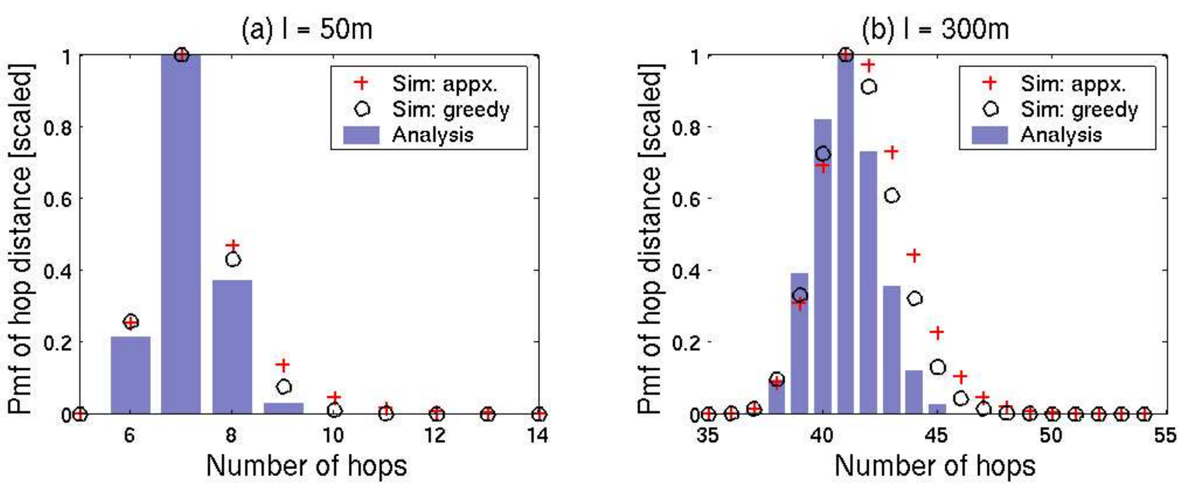

Figure 4: Distribution of number of hops for a low node density. $n = 5$.

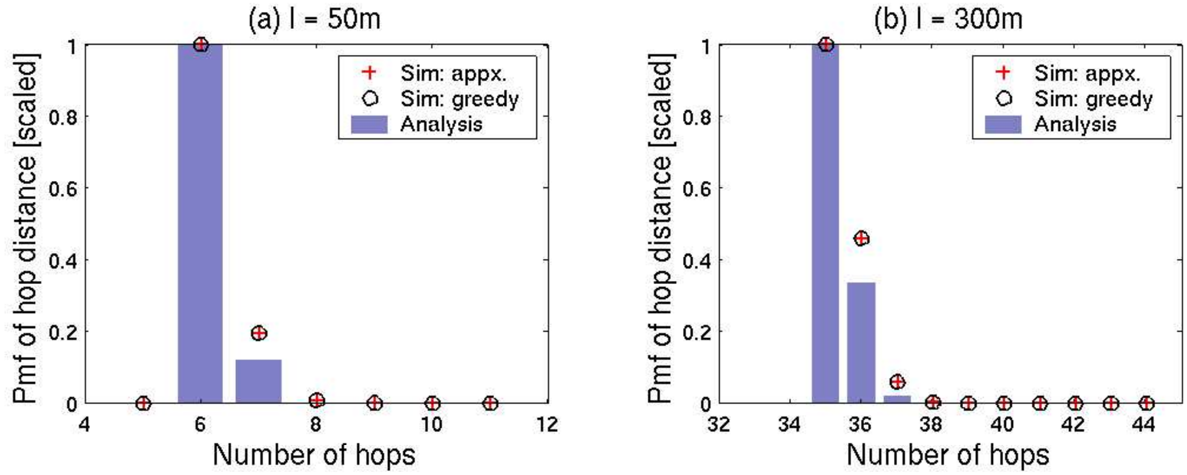

Figure 5: Distribution of number of hops for a given higher node density. $n = 15$.

In the simulation results, 'Sim:appx' corresponds to the analytic approximation of the forwarding area, and 'Sim:greedy' corresponds to the actual greedy forwarding. The simulation results have a good match with the analysis except for a little slower decay in the probability mass on the higher side of number of hops and a little faster decay in probability mass on the lower side of number of hops. This could be mainly due to the border effects in the simulation, which is not captured in the analysis. Although the source and destination nodes are chosen well inside the boundary, the intermediate nodes along the route may hit the boundary and the potential forwarding region for those nodes is less than that assumed in the analysis. For the border nodes, the chance of finding the best possible forwarding node is less than that without the border effect. Therefore, a route constituted by the border nodes may be longer. Another possible reason of minor discrepancy between the analysis and simulation results could be that in the analysis the chance of finding a forwarding node is assumed independent of previous hop. However, in reality (in the simulation), there is a one hop memory, so the chance of finding a forwarding node at one stage is partly determined by the process in the previous step. Also notice from Figs. 4 and 5 that this border effect along the route becomes more dominant as the source-to-destination distance increases and node density decreases.

Numerically obtained bounds on number of hops along with the mean for different source-to-destination distances and different node densities are shown in Fig. 6. Note that the difference between two bounds becomes lesser with the increase in node density. Also, the lower bound is quite tighter than the deterministic 'ideal' lower limit $\lceil \frac{l}{R} \rceil$.

4.3 Effect of analytic approximation

We note that greedy forwarding should ideally approach to reduce the remaining distance to the destination node. Referring to Fig. 1, this can be ensured only if the forwarding node lies within the lightly shaded region. However for simplicity in the analysis we have considered the greedy forwarding zone to be the half circle (the entire shaded region). Fig. 7 captures the error (cf. (3.12)) due to approximating the area of greedy forwarding in the analysis. It is noted that the error becomes increasingly insignificant as the node density increases. This observation is also corroborated with the supporting result in Figs. 4 and 5, where it is found that with low node density the simulated pmf plot with the area approximation ('Sim:appx')

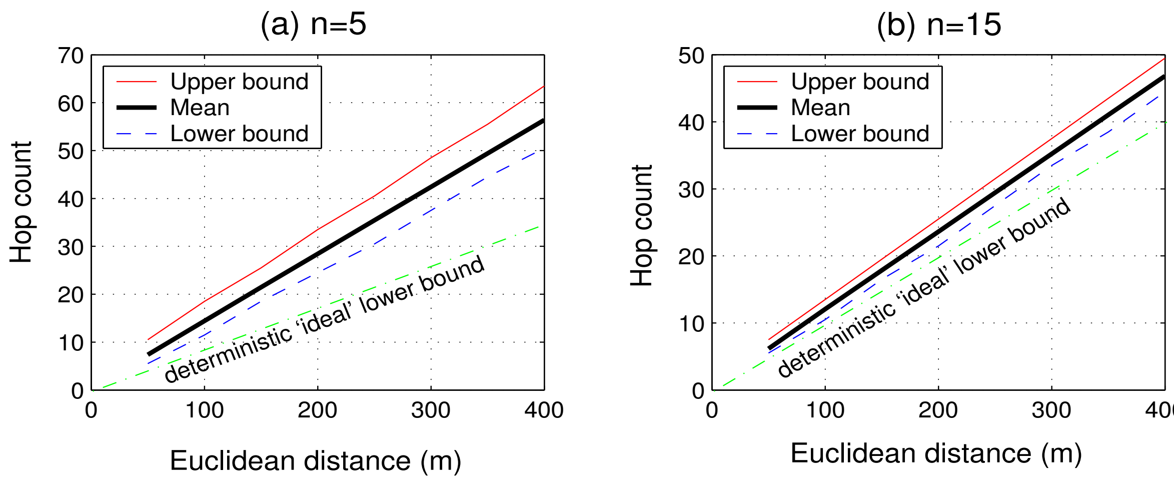

Figure 6: Bounds on hop distance.

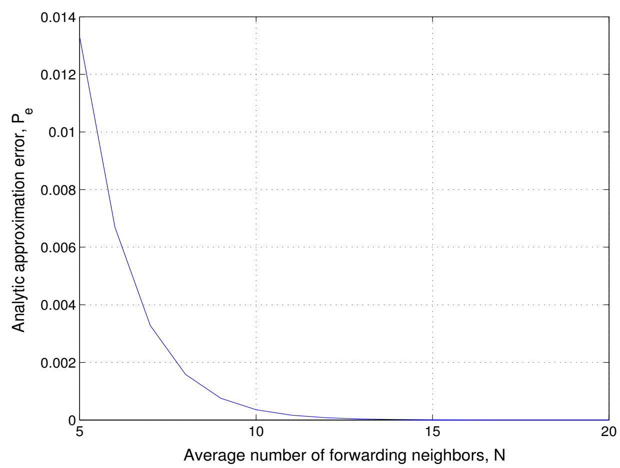

Figure 7: Effect of node density on the error in greedy packet forwarding.

does not match well with the pmf of actual greedy forwarding ('Sim:greedy').

4.4 Bounds on Euclidean distance from hop count

From the analytic expression in (3.8) one can also compute the bounds on Euclidean distance from a given hop distance. Numerically obtained results plotted in Fig. 8 shows how the tightness of bounds varies with node density for different network size. It also shows that for a given network size and a given node density, one can set a maximum limit in distance ambiguity while estimating the distance between any two nodes from the known hop count between them. For example, if the average number of forwarding neighbors is $n = 15$ (the corresponding node density is $\rho = \frac{2n}{\pi R^2}$) and the maximum hop count between any two nodes in the network is 30, the maximum ambiguity in distance estimation is bounded to three times the nodal range. As we will see in the next section (also shown in Fig. 9), this bound enables one to estimate the error in determining a node's position from the information on hop counts from the anchor nodes.

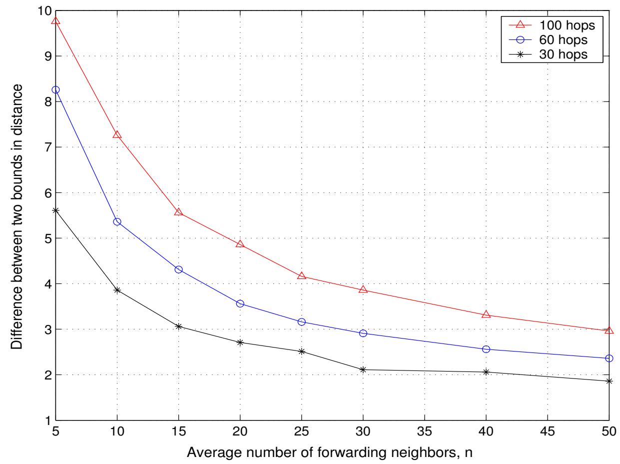

Figure 8: Variation of difference between lower and upper bounds (in unit of nodal range, R) with node density.

5 Potential Applications

An idea of the relation between hop count and the Euclidean distance may lead to the understanding to several networking aspects in ad hoc multihop networks. In this section we will briefly outline a few such areas.

5.1 Node localization in sensor networks

In an ad hoc multihop network, the knowledge of relative positions of nodes is very useful in delivering message from one to the other at a low network cost. As recently studied in [12], the field nodes can learn their virtual position information in terms of hop counts from the three elected anchors. This information is in turn used for routing between two geographically separated nodes. We note that from the known relationship between hop count and the Euclidean distance, it is also possible to obtain an approximate relative geographic location of a field node with respect to the anchor nodes' positions via standard trilateration approach [24]. More precisely, if a field node knows its distance in hop count from an anchor, the corresponding Euclidean distance can be estimated. The estimated distances of the field node from the tree anchors can then be used to calculate its approximate relative location via trilateration approach. Considering the bounds in Euclidean distance, the estimated position of the node will have a zone of ambiguity, as depicted by the shaded region in 9. Additionally, if the actual geographic locations of the anchor nodes are also known, an estimate of a field node's actual geographic location can also be derived – the estimation error being a function of the node density. As it is clear from Figs. 8 and 9, the zone of ambiguity becomes smaller with increased node density.

5.2 Delay and delay jitter

End-to-end delay is an important quality-of-service parameter (QoS) in QoS applications like multimedia traffic, and lowering the delay is one of the motivating factors in adapting greedy routing. Likewise, delay jitter is also a critical parameter in real-time applications. Having a

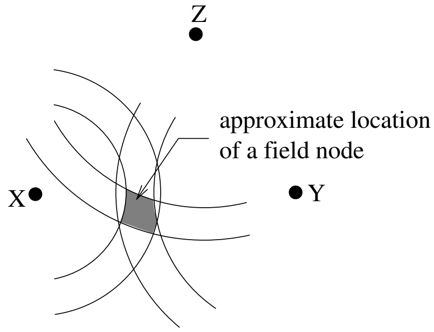

Figure 9: Estimation of a field node's location from the anchor nodes' positions. Two concentric arcs corresponding to an anchor node correspond to the bound in Euclidean distance of a node for a given (known) hop count. The distance between the two concentric arcs is the maximum bound for a given node density and a network size, and is approximately the same for any hop count within a network.

knowledge of average hop count between two nodes at a given Euclidean distance and an idea of per-hop trans-receive latency can enable one to estimate the end-to-end delay. In practical cases, the effects of network congestion and node failure rate can be added on top of this basic estimate. The bounds on hop count on the other hand helps evaluating the associated delay jitter. Depending on the nature of applications, the delay jitter can be controlled as desired by suitably altering the network density.

If the average Euclidean distance between any two communicating nodes and the average number of simultaneous communications in a network is known, from the relationship of a given Euclidean distance and the hop count the network traffic can be estimated.

5.3 Power consumption

Another important parameter determining the network lifetime in wireless ad hoc networks is the power consumption of a node. For a given signal-to-interference-and-noise ratio (SINR), the transmit power consumption is a function of transmitter-to-receiver distance, path loss factor, and activity of the neighboring nodes. The characteristic of per-hop progress in greedy routing gives the variation of transmitter-receiver distance. Based on the characteristic of one-hop progress, from the knowledge of power decay law, and for a given channel condition, transmit power consumption at a node can be estimated.

6 Conclusion

In this paper, we presented an analytical approach to compute the bounds on number of hops in greedy forwarding from a given source-to-destination Euclidean distance in a multihop network with uniformly distributed nodes. We obtained the characteristics of least remaining distance in greedy forwarding and showed that the average distance progress per hop is a function of node density and slowly varies with respect to current distance to the destination node. From this observation, a simple approximate approach was provided to obtain the average hop count. From the distribution characteristics of per-hop progress towards the destination, the lower and upper bounds on hop count were obtained via numerical simulation. We also showed that one can numerically compute the bounds on Euclidean distance from a given hop count. Our observations from analysis and numerical simulations were verified through network simulations.

Although our analysis was focused on greedy routing, it can be extended to a general routing approach where data forwarding is not restricted to a node closest to the destination. We anticipate that our approach to evaluating bounds on hop count could be useful in estimating important network parameters like end-to-end delay, delay jitter, traffic load, transmit power consumption, etc., and in developing virtual coordinate based node localization.

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