Edge-Preserving Permittivity Range Profile Reconstruction by a Genetic Algorithm

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Abstract

Reconstructing the permittivity range profile of a layered medium from bandlimited backscattered data always implies either higly unstable or oversmoothed results. To avoid this drawback, we introduce an edge-preserving regularization constraint with explicitly marked lines. The inverse scattering problem is solved by a genetic optimization algorithm.

Keywords: Microwave tomography, range profile reconstruction, edge-preserving regularization, genetic algorithms.

1 Introduction

Microwave tomography has not been finding but a very few practical applications in the past decades, because of specific difficulties that made it much more difficult to implement than, say, X-ray tomography. These difficulties are related to both the nonlinear nature of the direct scattering problem and the design of efficient measurement systems to be used in field-applications.

On the other hand, microwaves, and electromagnetic fields in general, could offer unique advantages over other probing radiations, such as X-rays or ultrasounds, since they can be treated by simple and unexpensive hardware, and show good sensitivities to important parameters characterizing various kinds of diagnostic applications.

Using a fully nonlinear data model would allow very accurate algorithms to be derived for quantitative electromagnetic tomography. Since the late 1970s, the complete nonlinear scattering equations have been embedded into optimization strategies for tomographic reconstruction based on matching the measured

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data with the ones synthesized from the estimated object function. However, this requirement alone does not yield good results, since the electromagnetic scattering inversion problem is strongly ill-posed. The classical Tikhonov regularization [9] cannot completely solve the problem, since it usually leads to oversmoothed solutions. On the other hand, all the techniques proposed to avoid this inconvenience are very expensive computationally, and only recently they are becoming applicable to problems of practical interest.

In this paper, we consider the problem of reconstructing the dielectric range profile of a lossless layered object on the basis of the complex reflection coefficient measured within a certain frequency band. This is significant in different nondestructive diagnosis fields, such as in preservation of cultural heritage or geophysical inversion, and has already been adressed by several authors [1] [3] [4] [6] [7] [10]. Our approach is to match synthetic to measured data, regularizing the solution my means of an edge-preserving local smoothness term [3]. Optimization is performed by means of a genetic algorithm, which is able to avoid possible local optima in the fitness function. The advantages we expect from this approach are on accuracy, by virtue of the very accurate forward solver and the edge preserving regularization, which should guarantee a good spatial resolution.

The paper is organized as follows. In Section II, we introduce the problem of range profile reconstruction, briefly describing our particular solution strategy. Samples of simulated experiments are shown in Section III. Concluding remarks and possible future developments are drawn in Section IV.

2 1D inversion of backscattering data

Consider the geometry shown in Figure 1. A lossless dielectric wall of thickness L, surrounded by air, is probed by an electromagnetic plane wave impinging on the wall at normal incidence and with a frequency belonging to the microwave range. The wall has infinite extension in the transverse directions, ans its dielectric properties only depend on the depth coordinate z. The complex reflection coefficient $\rho(f)$ is measured at N_f frequencies within a certain bandwidth.

The inverse problem of permittivity range profile reconstruction consists in finding an estimate of the real permittivity function $\epsilon(z)$ from the measured values of $\rho(f)$, which can be arranged in a complex N_f-vector ρ_{meas}.

If, for any $\epsilon(z)$, we are able to evaluate an N_f-vector $\rho_{calc}(\epsilon)$, we can implement an optimization procedure for profile reconstruction, based on data fitness and regularization. To solve the above problem, we discretize it by dividing the wall into a finite number N of homogeneous layers of thickness L/N. Our unkonwn thus reduces to a real vector of N elements, where the generic entries represent the permittvities of the corresponding layers.

The forward solver we use to evaluate $\rho_{calc}(\epsilon)$ [11] analyzes the structure in Figure 1 as a cascade of transmission line segments of length L/N and characteristic admittances $\omega\epsilon_n/\beta_n$, where ω is the working angular frequency, ϵ_n is the permittivity of the n-th layer, and β_n is the propagation constant inside an

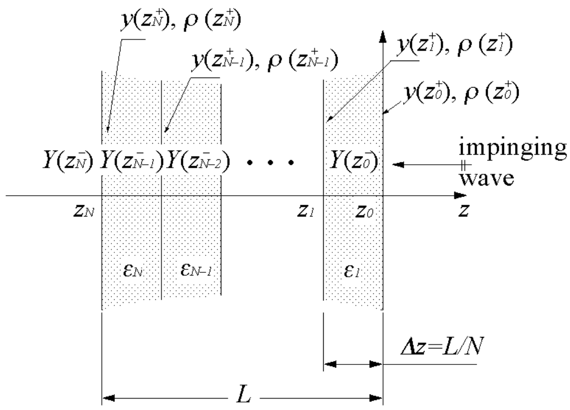

Figure 1: Measurement setup and problem discretization.

infinite medium with the same characteristics as the one filling the n-th layer at the operating frequency ω. The reflection coefficients are found by evaluating the admittance of the entire cascade of equivalent transmission lines seen by the wave impinging at the interface $z = 0$. Let β_o be the propagation constant in free space at the frequency ω. The electric field incident at $z = 0$ is denoted by $E^{inc} = E_o e^{j\beta_o z}$. We initialize the admittance $Y(z_N^-)$ seen at the interface $z = z_N$ towards the negative z axis at the value of the characteristic admittance of free space:

$$Y(z_N^-) = \omega\epsilon_o/\beta_o \tag{1}$$

Then, for each interface at $z = z_n$, up to $z_n = 0$, we repeat the following steps.

- Compute the normalized admittance seen through the interface towards the negative z axis:

$$y(z_n^+) = Y(z_n^-)/(\omega\epsilon_n/\beta_n) \tag{2}$$

- Compute the reflection coefficient $\rho(z_n^+)$ at the interface $z = z_n$

$$\rho(z_n^+) = [1 - y(z_n^+)]/[1 + y(z_n^+)] \tag{3}$$

- Rotate the reflection coefficient to the next interface:

$$\rho(z_{n-1}^-) = \rho(z_n^+)e^{-j\beta_n(z_{n-1}-z_n)} \tag{4}$$

- Compute the unnormalized admittance at the next interface:

$$Y(z_{n-1}^-) = \omega\epsilon_n[1 - \rho(z_{n-1}^-)]/\{\beta_n[1 + \rho(z_{n-1}^-)]\} \tag{5}$$

The whole procedure is then repeated for all the frequencies. At the end of this process, the vector $\rho_{calc}(\epsilon)$ is obtained.

For ϵ to be a good estimate of the actual permittivity profile, it should produce a good match between $\rho_{calc}(\epsilon)$ and the vector ρ_{meas}. Unfortunately, this is not enough to determine a robust solution to an inverse problem. A regularization strategy must be adopted [9]. This normally requires a certain degree of smoothness to be enforced into the solution. However, when the true solution has significant discontinuities, enforcing global smoothness is often not appropriate, since the edges, which contain essential information on the object being tested, are lost in the reconstruction. Among the techniques used to enforce local smoothness, that is, able to break the smoothness constraint where edges are present, there is the introduction of an explicit *line process* in the regularization function [3]. By this approach, our inverse problem entails the optimization of the following functional:

$$F(\epsilon) = d(\rho_{meas}, \rho_{calc}(\epsilon)) + \lambda \sum_{k=1}^{N-1} (\epsilon_{k+1} - \epsilon_k)^2 (1 - l_k) + \alpha l_k \tag{6}$$

where d is a suitable distance between the measured and the calculated vectors. The supplementary variables l_k appearing in Equation (6) are intended to preserve abrupt permittivity variations where these are likely to occur. Each l_k is a binary variable located between the k-th and the $(k+1)$-th cells. Observe that, when $l_k = 1$, the regularization functional does not penalize any difference between the values of ϵ_k and ϵ_{k+1}, that is, the contribution of these two layers to the functional is constant, its value being α. This means that the presence of any discontinuity between layers k and $k+1$ is paid by a constant contribution to the fitness functional. Conversely, when $l_k = 0$, the difference between ϵ_k and ϵ_{k+1} is quadratically penalized, and a significant discontinuity is not allowed. It is easy to see that the quantity $\sqrt{\alpha/\lambda}$ acts as a threshold for allowing a discontinuity to be introduced.

Owing to the presence of the line process, functional (6) is nonconvex, and a global optimization algorithm is required. Normally, the classical techniques are not suitable to solve nonconvex problems, since they may get stuck on local extrema. This difficulty can be faced by both deterministic and stochastic strategies. The genetic approach [2] consists in a class of nondeterministic algorithms that have already been proposed for electromagnetic scattering inversion [4] with global smoothness or no regularization at all. Our aim here is to test the performance of a genetic algorithm in minimizing the functional in Eq. (6). Unlike other stochastic approaches, a genetic algorithm does not need to sample any probability density related to the objective function. Instead, it mimicks what is supposed to happen in biological evolution: the best fitness is reached by a sort of *natural selection*.

In our implementation [5], the process starts by creating a very large set of binary-coded trial solutions (called *chromosomes*) for the proposed problem. The relevant parameters are a set of N real permittivities plus a set of $N-1$ binary line elements. The chromosomes are organized into populations of equal size. The population with the fittest chromosomes (that is, the ones that provide the best values for the fitness functional) is selected to form the

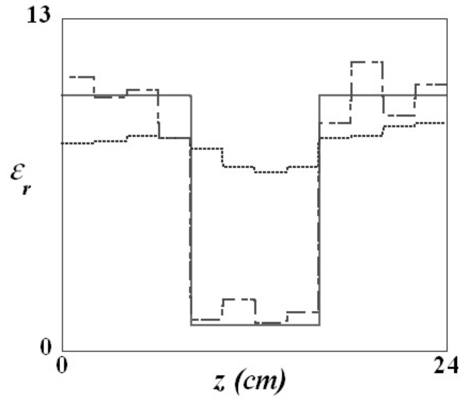

Figure 2: Reconstruction of a simulated discontinuous profile from 25 dB-SNR data. Original (solid line); reconstructed, global smoothness, $\lambda = 50$ (dotted line); reconstructed, edge preserving, $\lambda = 3.5$, $\alpha = 56$ (chain line).

first generation. A uniform crossover operator is used to breed new individuals from randomly chosen couples of chromosomes. The possibility of exploring the solution space is further enhanced by a subsequent *mutation operator*, which consists in randomly switching one bit in each chromosome string. The mutation probability is determined adaptively on the variations in the best fitness. In order to let evolution proceed, each chromosome is assigned with a fitness-induced selection probability. Then, a new population is generated by a modified roulette wheel selection. Finally, if the best solution of the old population is better than the best of the new one, the worst individual of the new generation is replaced by the best of the old one. This is called *simple elitism*, and, along with the particular mutation strategy, has been proved to ensure convergence to a global optimum [8]. If this new stage of evolution does not contain a solution which satisfies the criterion of minimum fitness required, the loop restarts with this new generation; otherwise, the algorithm stops and the process ends.

3 Some results

Samples of preliminary numerical results are shown in this section to demonstrate the effectiveness of the method proposed, when different levels of measurement noise are superimposed to the signal. Some of our experiments are devoted to assess the advantages of using the line process as compared to the globally smooth solutions. As an example, in Figure 2 we show an original profile, simulating an indefinite concrete wall ($\epsilon_r = 10$) of thickness 24 cm with an air inclusion inside. We solved the problem by using $N = 12$ layers and the reflection coefficient values for $N_f = 100$ frequencies in the range 0.8 GHz - 3 GHz, assuming a 25 dB SNR. The figure also shows the profiles reconstructed by the global smoothness and the edge-preserving regularizers. These results were obtained after about 600,000 generations. As can be seen from the figure, the introduction of the line process allows the profile discontinuities to be preserved, while they are lost in the globally smooth solution. All our present results confirm these features, and are obtained by considering different dielectric profiles and noise levels (with SNRs in the range 10 dB to 30 dB). The frequency range, instead, has been kept fixed. The experiments to evaluate the effective spatial resolution are about to be completed. The next step will be to test the method with more realistic data. This will allow us to assess more accurately the performance still being able to compare the results to ground truth. In a successive phase, we will apply our technique to real data. Further developments will be devoted to improve the optimization efficiency. Genetic algorithms, indeed, are normally very expensive, but they naturally allow several forms of parallelism to be exploited. After that, we will be able to apply our strategy to the reconstruction of 2D permittivity distributions.

4 Conclusion

A technique to invert multifrequency backscattering data from a discontinuous permittivity profile has been presented. Explicitly marking the discontinuity locations is an advantage over global smoothness regularization, but prevents classical optimization algorithms from being used, since they are not able to avoid local optima. We chose to use a genetic algorithm to this purpose. The preliminary experimentation is providing promising results.

Important applications of this technique can be envisaged in different fields, such as masonry structures diagnosis, ground penetrating radar signal processing, seismic or electromagnetic oil reservoir prospection.

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