

The emergence of self-organization in complex systems - Preface

Paolo Paradisi^{1,2}, Giorgio Kaniadakis³, Antonio Maria Scarfone⁴

¹*Istituto di Scienza e Tecnologie dell'Informazione "A. Faedo" (ISTI-CNR), Via Moruzzi 1, 56124 Pisa, Italy*

²*BCAM - Basque Center for Applied Mathematics, Alameda de Mazarredo, 14 E-48009 Bilbao, Basque Country, Spain*

³*Dipartimento di Scienza Applicata e Tecnologia (DISAT), Politecnico di Torino, Corso Duca degli Abruzzi 24, 10129 Torino, Italy*

⁴*Istituto dei Sistemi Complessi (ISC-CNR), c/o Politecnico di Torino, Corso Duca degli Abruzzi 24, 10129 Torino, Italy*

1. Complexity as emergence of self-organization

The interest towards self-organized and cooperative systems has rapidly increased in recent years. This is largely due to the recent availability of large data-sets where the links among data can be easily described in terms of a network with complex link topology and structure. Important examples of complex networks are social networks (e.g., the web) and the -omics emerging in several research fields such as biology (proteomics, genomics, metabolic networks), neurophysiology (connectomics) and physics (condensed matter). This has triggered an increasing interest towards the dynamics of complex networks. This hot research field is sometimes denoted as *complexity science*.

Actually, a definition of a *complex system* that is universally accepted by the scientific community does not yet exist, but there are some basic features that are recognized to characterize complex systems. Firstly, a complex system is multi-component, i.e. it is composed of many degrees of freedom: many individuals, particles, units or, in general, many sub-systems that are embedded in a network of strong *nonlinear* interactions. A complex system is often described as a network with a given set of nodes that interact by means of a set of links with complex topology. This aspect explains why statistical physics and network science have been the first research fields with a rapidly increasing interest towards complexity.

It is worth noting that the presence of many degrees of freedom and nonlinearity is not sufficient to get a complex behaviour. In a complex system the nonlinear dynamics must be cooperative, thus giving rise to the so-called emergent properties, and this seems to be the most peculiar and crucial feature characterizing complexity. Emergent properties are associated with the emergence of self-organized or coherent structures. These structures are states of the system whose temporal and spatial scale spectra (or, equivalently, long-range time-space correlations) cannot be represented as a simple function of external forcing or derived by the microscopic dynamics through some coarse graining procedure. In other words, it is not possible to link the large temporal, spatial and/or topological scales of self-organized structures with the correspondent small scales of the micro-dynamics. In summary, a multi-component system can be considered "complex" when its dynamics trigger the emergence of self-organized structures, so that the typical approach to complexity is focused on identifying self-organized struc-

tures, on their analysis and description, and on the modeling of their dynamical evolution.

Power-law, anomalous diffusion and intermittency in self-organization

A system that can be decomposed into independent systems, which is the opposite concept of self-organization, is characterized by exponential behavior, or sum of exponentials. Then, self-organization is expected to be associated with non-exponential behavior. More precisely, the self-organizing mechanism is usually related to a scale-free condition determining the emergence of self-similarity and fractality. For this reason, the most ubiquitous features found in self-organized complex systems are given by: power-law behavior, e.g., long-range time and/or space correlations, scale-free distribution of some quantity such as the average degree of nodes in a complex network, avalanche size in self-organized critical systems, cluster size in percolation. A fundamental aspect of complex systems is related to the transport properties, which are usually characterized by anomalous scaling, i.e., non-linear time dependence in the growth of the variance. This condition is also known as *anomalous diffusion*. Another aspect often found in complexity is a property known as *metastability*. In fact, emerging self-organized, long-lived, structures are often metastable and characterized by a sequence of short-time events marking their birth and death. Consequently, the dynamics of this class of complex systems are characterized by a birth-death intermittent process of cooperation, which is modeled through a renewal point process [1]. This important property is often found in biological complex systems and it is characterized by a fractal (inverse power-law) distribution of the inter-event times (see, e.g. [2, 3, 4, 5]).

2. Complexity: a survey of recent findings and modeling approaches

Without any claim of being complete, this Special Issue is a collection of papers giving a brief account of some recent findings and modeling approaches in the field of complexity science.

Stochastic modeling and noise in complexity

In papers [13, 14, 15, 16, 17] the modeling approach to complexity is based on the theory of stochastic processes. All these

papers, with the exception of the last one, are based on the standard or generalized Langevin equation. Depending on the specific application, the noise terms can be interpreted as the effect of self-organization, of an external environment or of thermal fluctuations. In the first case the noise term is typically long-range correlated. In some of the proposed models the authors also study the combined effect of different noise sources.

In paper [13] the role of noise in three different nonlinear relaxation processes is discussed. In particular, the authors give a brief review of noise-induced phenomena in far-from-equilibrium systems, with particular attention to condensed matter complex systems. In one of the considered systems the dynamics, given by a perturbed stochastic sine-Gordon equation, is characterized by switching events between a superconducting metastable and a resistive state.

In paper [14] the authors study the application of a multi-particle Monte Carlo modeling approach to the non-linear transport of electrons inside a semiconductor bulk. The fluctuating electric field is modeled as a Gaussian correlated noise, triggering the emergence of cooperative behaviour of electrons, which self-organize among different valleys. This has the counter-intuitive effect of reducing the intrinsic noise. The authors underline the interplay among internal time scales, related to electron dynamical response, and external time scales, related to the fluctuating electric field, in the emergence of complex self-organized structures.

An ubiquitous feature observed in many complex systems is power-law statistics. This aspect is theoretically investigated in paper [15] by means of a stochastic modeling approach. In order to describe anomalous diffusion occurring in complex dynamical systems, the authors investigate a nonlinear Langevin model driven by a Lévy stable noise. This model is shown to generate signals exhibiting power-law decay in statistical properties such as the steady state probability distribution and the $1/f^\beta$ power spectrum.

In paper [16] a “generalized Langevin equation with a power-law-type memory kernel is used to model the complex structure of the viscoelastic media”. The superposition of different kind of noise terms is studied: a multiplicative white Gaussian noise, an internal fractional Gaussian noise and an additive external white noise. This work is an interesting example of how a stochastic force (noise) can modify the behaviour of nonequilibrium complex systems in a counter-intuitive way. In particular, the authors investigate the interplay of memory and of the shear fluid flow on the cross-correlation functions of harmonically trapped Brownian particles, finding the important effect of memory-induced sign reversals.

The authors of paper [17] discuss the effects of a Poisson point process, which is seen as a sort of additive “noise” term, on a complex signal characterized by fractal intermittency. The complex signal is represented as a telegraph signal, i.e., a dichotomous signal that can assume only two states, but with a power-law distribution of transition times. When considering the diffusion process associated with this intermittent signal, long-time normal diffusion is found even if the complex signal would generate anomalous diffusion, but a signature of the underlying complex signals is seen in the scaling of the diffusivity

coefficient with the exponent of the time distribution.

Complexity in heterogeneous systems

Papers [18, 19, 20, 21, 22, 23] are mainly devoted to anomalous transport in complex media and complex materials, from complex fluids to porous media. In these systems complexity is intimately connected with some physical properties that are heterogeneously distributed, even if, in some applications, the medium can be considered statistically homogeneous (see paper [20]).

The authors of paper [18] discuss the self-organizing character of two-dimensional turbulence by using a stochastic Lagrangian approach. They show that self-organization is associated with trajectory trapping, being a typical particle trajectory characterized by a random sequence of long (non-Gaussian) jumps and trapping events. This is intimately connected to the anomalous behaviour of transport coefficients. Anomalous transport and particle trapping are associated with the well-known inverse energy cascade (from small to large scales), which determine the emergence of large scale quasi-coherent vortices, being coherence identified by the presence of long range correlation with slow power-law decay.

Paper [19] is devoted to the same problem, but here the authors investigate the self-organizing of the vorticity field by comparing a statistical approach with a field-theoretical formulation. In both papers it is explained that the self-organizing mechanism, intimately connected to the inverse energy cascade, is given by the attraction of large scale vortices on the small scale vortices of the same sign (clusterization of like-sign vorticity), thus generating a sort of coalescence dynamics in the vorticity field.

Paper [20] illustrates a model of impurity transport in a porous medium. The authors make evident the role of the detailed non-homogeneous structure of the medium, even if statistically homogeneous, in the emergence of power-law behaviour, scaling and anomalous transport. This work is an interesting example of complex (power-law) behaviour determined by the heterogeneous structure of an underlying complex medium where particle transport takes place. In these systems, local transport is normal and given by standard advection and diffusion, while the global observed behaviour is a sum over all the heterogeneous regions.

It is worth mentioning that a similar approach, i.e., the random slow modulation of a parameter due to a non-homogeneous, disordered environment, is the basis of the well-known superstatistics [10, 11], where the global statistics is derived by a linear average computed over the parameter distribution. If complexity should be identified with the emergence of self-organized structures, it could be questionable if heterogeneous systems should be considered complex. In our opinion, they could be included considering that the underlying complex structure of the medium has been generated by some cooperative dynamics and is typically characterized by fractal properties. In this sense, the fractal heterogeneity of the medium is a manifestation of complexity triggering anomalous scaling in the transport properties.

Superstatistics was proposed to be a rigorous way to derive Tsallis statistics from a dynamical model [12]. The Tsallis dis-

tribution is in fact the main topic of paper [21], where the authors discuss the ubiquitous presence of quasi-power law distributions in multiparticle production processes. In fact, the Tsallis distribution is essentially an inverse power-law function for large values and it was derived in the framework of a non-equilibrium statistical model involving the minimization of a non-extensive entropy (see discussion and references in paper [21]). The Tsallis entropy and distribution represent an attempt to build a non-standard statistical mechanics for complex systems, and superstatistics proved to be compatible with systems whose complexity originates from non-homogeneity of some physical parameter in the medium supporting the anomalous transport. It is not yet clear if this picture is in agreement with other modeling approaches, such as the fractional one (see [8, 9] and the comment to papers [24, 25] reported below). Paper [22] illustrates the application of kinetic theory and hydrodynamic equations to a granular binary mixture, which is an interesting prototype of complexity. In fact, granular fluid flows are not only “composed of many degrees of freedom (many particles) that are embedded in a network of strong nonlinear interactions”, but also characterized by the emergence of self-organized structures (velocity vortices and density clusters). Interestingly, the authors underline the central role of instabilities and of transport coefficients in the self-organizing mechanism. A zeta urn model following a quench from high temperature to a final state with temperature below the condensation one is studied in paper [23]. This is an example of an aging system displaying scaling behaviour. In this case an equilibrium state exists, but the relaxation process is so slow that the system evolves without reaching it in a finite time. This condition is found in many complex materials, such as ferromagnets or complex fluids and glasses, and characterizes the response to an abrupt change, caused by an external perturbation, in the control parameters. This paper illustrates an interesting aspect of complexity, which is relative to the dichotomy equilibrium vs. non-equilibrium. In the investigated model the cooperative dynamics determine the separation of degrees of freedom into two different groups, the fast one and the slow one. The first group is responsible for the observed quasi-equilibrium features, while the second one is associated with aging. This separation triggers the emergence of interesting scaling behaviours in the physical observables.

Fractional operators as emergent dynamics in complex systems

In papers [24] and [25] the complex behaviour is associated with the emergence of fractional derivative operators [6]. These two papers discuss two alternative approaches in the derivation and interpretation of fractional operators: in [24] the trapping mechanism is modeled, while, in [25], the heterogeneity of the system is considered.

In paper [24] the authors study “the kinetics of subdiffusion-limited growth and dissolution of nanoprecipitates in disordered solids on the base of subdiffusion equations with fractional derivatives” and compare their analytical results with Monte Carlo stochastic simulations. The emergence of fractional dynamics is here related to the formation of self-organizing clusters in the system, which determine the anomalous transport

and diffusion properties of impurities and defects in disordered solids, which have a complex heterogeneous structure and contain impurities. At variance with ordered crystalline structures, in disordered media random fluctuations affect transport phenomena, a condition often modeled by means of the distribution of some parameter (e.g., relaxation and/or diffusivity parameters).

The authors follow a modeling approach based on trapping mechanisms and, thus, they apply the well-known Continuous Time Random Walk (CTRW) to derive the fractional equations describing anomalous sub-diffusion. [7].

In paper [25] the topological complexity is explicitly investigated by considering the interplay between structure and dynamics in semiflexible, hierarchically-built fractal polymers under external pulling forces. A Langevin modeling approach is here used with the introduction of a random relaxation parameter. Under the assumption of a power-law distribution of the parameter, related to the topological structure of the polymer, the authors rigorously prove that the considered dynamical quantities are described through equations involving fractional calculus operators, in agreement with former phenomenological fractional laws in polymer physics.

This is an interesting example of how a complex structure or a complex medium can determine the emergence of fractional calculus and anomalous diffusion. This result is even more interesting as it has been depicted in the framework of a complex biological system. In recent years, the derivation of fractional operators in the context of disordered, non-homogeneous media has attracted some interest (see, e.g., [8, 9]), and paper [25] surely gives an important contribution in shedding light into the relationship between complexity and fractional calculus.

Complexity in biological systems

Papers [26, 27, 28, 29, 30, 31] are focused on the investigation of biological systems. Biology is probably the most intriguing research field from the point of view of complexity science. In fact, the first requirement for a biological system to be alive, and to survive, is the need for some kind of self-organization. In this sense, a biological system represents the best prototype of complexity. We note that also paper [25], already introduced above, could be included in the section dedicated to biological applications, but we decided to discuss it in the framework of fractional calculus and complexity due to heterogeneity.

The central role of fractality and self-similarity is discussed in paper [26], being the study focused on the role of geometrical structures. Clearly, fractality is strictly connected to self-organization as a fractal object is generated through a cooperative dynamics and the transport over a geometrical fractal is typically anomalous, displaying power-law and scaling. The authors find the emergence of three-dimensional structures in two-dimensional monomolecular layers of fatty acids, thus characterizing the self-similarity of the system by means of fractal dimensions. In these systems many physical properties, such as the specific capacitance, depend not only on the material, but also on the fractality of the structures and, thus, on the kind of nonlinear dynamics among the different components of the system, which are given, in this case, by the nucleation dynamics.

In paper [27] the authors discuss an interesting approach, based on Random Matrix Theory, to the modeling of the dynamics of RNA three-dimensional conformational structure. The non-linear random matrix model is used to simulate the RNA folding and, in particular, the transitions among different conformational states, giving a good agreement with the analysis of real data carried out in the paper. Thus, the complexity of the system is described by means of the topological features of the system. Even if the modeling approach is different, the underlying qualitative idea of this paper is similar to that discussed in paper [25].

In paper [28] a biophysical phenomenon known as Quorum Sensing (QS) is studied. QS is a very interesting example of social collective behaviour in bacteria, being the nonlinear interactions given here by the secretion and detection of some chemical signaling molecules or, as in this paper, of bioluminescence signals that are generated by bacteria through a chemical reaction. The emergence of self-organizing behaviour is interpreted as an ecological strategy providing more opportunities for the group survival. A crucial role is assumed by the statistical distribution of the inter-event times, being each event given by the bioluminescent emission. This approach lies into the general framework of complexity interpreted as *fractal intermittency*, also denoted as *Temporal Complexity* and described in papers [17] and [31].

The authors of paper [29] investigate the diffusion properties of a system of active dumbbell molecules with repulsive interactions, which is a prototype of Active Matter. In Active Matter the particles are self-propelled units that are not passively advected by the fluid flow. Active Matter is characterized by a "continuous partial conversion of internal energy into work". As underlined by the same authors, "these systems live, or function, in conditions far from thermodynamic equilibrium and pose challenging questions to non-equilibrium statistical mechanics" and "exhibit non-trivial collective properties".

The authors of paper [30] illustrate an interesting neural model that is inspired to the physiological sleep-wake cycle in the brain. They discuss the constructive role of non-homogeneity and disorder in complex systems. To this goal, they build a network of Hodgkin-Huxley-type neurons that is inspired to the real physiology of the brain, even if the heterogeneity is limited to only two different kind of neurons. This means that their modeling approach is focused on the microscopic level of the single neuron dynamics, but it is able to reproduce, by means of emergence of global self-organizing behaviours, the sleep-wake cycle. In particular, they show how emergent behaviour and, in particular, the sleep-wake cycle, is related to the interplay between heterogeneous units in the neural network, which is an example of diversity-induced resonance. It is worth underlining the following sentence of the authors:

"The effect studied can be considered as a most typical example of global phenomenon emerging from the non-linear interaction of the units of a complex system, not only because it provides a physiological model of the complex behavior of a neuronal process but also because it is based on the presence of a suitable level of heterogeneity, a common property of complex systems that by definition can only be realized in systems composed of

many interacting units."

The last, but not least, contribution of this collection is given by paper [31]. This is a very nice discussion about a novel view about the analysis and modeling of complex biological systems, a hot research field that the author denotes as *Biological Complexity*. The proposed view is based on a combination of ideas coming from critical phenomena, renewal theory and long-term memory stochastic processes (e.g., Generalized Langevin Equation and Fractional Brownian Motion). The emergence of self-organization in biological systems is evident, but there's the lack of general leading principles, which is a very old problem of theoretical biophysics with respect to other field where the theoretical research can refer to guiding principles (e.g., the postulates of classical mechanics). The author rightly doubts that this reductionist point of view will never work in the case of complex systems and, in particular, in theoretical biology, as the emergence of self-organization cannot be reduced to a sum of microscopic, independent, units.

It is true that anomalous diffusion and scaling, as well as fractional dynamics, superstatistics or Tsallis distribution, can emerge from a linear superposition of effects generated by a non-homogeneous medium determining observed random fluctuations in some crucial parameters. However, on one hand, this approach is not able to explain the emergence of self-organized structures from strong nonlinear dynamics. On the other hand, it is based on a heterogeneity of the medium that must be complex itself in order to generate anomalous transport. Thus, in order to get this complex (fractal) heterogeneity, a previous self-organizing mechanism has to be assumed, but this again implies the existence of cooperative, nonlinearly interacting, units.

Acknowledgments We wish to thank the journal "Chaos, Solitons and Fractals" for hosting this special issue dedicated to complexity. We are extremely grateful to all the Editorial staff who have been very helpful in the realization of this issue. A special thank to all the contributors for their different manuscripts. Finally, we would like to thank gratefully all the anonymous referees who assisted us in the reviewing process.

References

- [1] D. R. Cox, *Renewal Theory*, Methuen & Co. Ltd., London, 1962.
- [2] P. Allegrini, D. Menicucci, R. Bedini, L. Fronzoni, A. Gemignani, P. Grigolini, B.J. West and P. Paradisi, Spontaneous brain activity as a source of ideal $1/f$ noise, *Phys. Rev. E* **80**, 061914 (2009).
- [3] P. Paradisi, P. Allegrini, A. Gemignani, M. Laurino, D. Menicucci, A. Piarulli, Scaling and intermittency of brain events as a manifestation of consciousness, *AIP Conf. Proc.* **1510**, 151-161 (2013).
- [4] P. Allegrini, P. Paradisi, D. Menicucci, M. Laurino, R. Bedini, A. Piarulli, A. Gemignani, Sleep unconsciousness and breakdown of serial critical intermittency: New vistas on the global workspace, *Chaos Soliton Fract.* **55** (2013) 3243.
- [5] P. Paradisi, D. Chiarugi and P. Allegrini, A renewal model for the emergence of anomalous solute crowding in liposomes, *BMC Syst. Biol.* **9**(Suppl 3): S7 (2015).
- [6] I. Podlubny, *Fractional Differential Equations*, Academic Press, New York (1998); H. Schiessel, C. Friedrich and A. Blumen, *Applications of Fractional Calculus in Physics*, World Scientific, Singapore (2000); F. Mainardi, *Fractional calculus and waves in linear viscoelasticity*, Imperial

College Press, London (2010); B.J. West, M. Bologna and P. Grigolini, Physics of fractal operators, Springer-Verlag, New York (2003).

[7] R. Metzler and J. Klafter, The random walks guide to anomalous diffusion: a fractional dynamics approach, *Phys. Rep.* **339**, 177 (2000).

[8] G. Pagnini, Short note on the emergence of fractional kinetics, *Phys. A* **409**, 29-34 (2014)

[9] P. Pramukul, A. Svenkeson, P. Grigolini, M. Bologna and B.J. West, Complexity and the fractional calculus, *Adv. Math. Phys.* **2013**, article ID 498789 (2013).

[10] C. Beck, G.S. Lewis and H.L. Swinney, Measuring nonextensivity parameters in a turbulent Couette-Taylor flow, *Phys. Rev. E* **63**(3II), 353031-353034 (2001).

[11] C. Beck, Generalized statistical mechanics for superstatistical systems, *Phil. Trans. Royal Soc. A: Math. Phys. Eng. Sci.* **369**(1935), 453-465 (2011).

[12] C. Beck, Dynamical foundations of nonextensive statistical mechanics, *Phys. Rev. Lett.* **87**(18), 180601 (2001).

[13] B. Spagnolo, D. Valenti, C. Guarcello, A. Carollo, D. Persano Adorno, S. Spezia, N. Pizzolato and B. Di Paola, Noise-induced effects in nonlinear relaxation of condensed matter systems, *Chaos Soliton Fract.* (2015), DOI:10.1016/j.chaos.2015.07.023

[14] D. Persano Adorno, N. Pizzolato, P. Alaimo, B. Spagnolo, B. Di Paola, Electron dynamical response in InP semiconductors driven by fluctuating electric fields, *Chaos Soliton Fract.* (2015), DOI:10.1016/j.chaos.2015.08.022

[15] R. Kazakevicius and J. Ruseckas, Power-law statistics from nonlinear stochastic differential equations driven by Levy stable noise, *Chaos Soliton Fract.* (2015), DOI: **DOI TO BE INSERTED**

[16] A. Sauga, K. Laas and R. Mankin, Memory-induced sign reversals of the spatial cross-correlation for particles in viscoelastic shear flows, *Chaos Soliton Fract.* (2015), DOI: **DOI TO BE INSERTED**

[17] P. Paradisi and P. Allegrini, Scaling law of diffusivity generated by a noisy telegraph signal with fractal intermittency, *Chaos Soliton Fract.* (2015), DOI:10.1016/j.chaos.2015.07.003

[18] M. Vlad and F. Spineanu, Trajectory statistics and turbulence evolution, *Chaos Soliton Fract.* (2015), DOI:10.1016/j.chaos.2015.08.023

[19] F. Spineanu and M. Vlad, Self-organization of the vorticity field in two-dimensional quasi-ideal fluids: The statistical and field-theoretical formulations, *Chaos Soliton Fract.* (2015), DOI:10.1016/j.chaos.2015.05.034

[20] V.A. Kutsepalova and L.V. Matveev, Non-classical regimes of colloid-facilitated impurity transport in statistically homogeneous double porosity media, *Chaos Soliton Fract.* (2015), DOI:10.1016/j.chaos.2015.07.024

[21] G. Wilk and Z. Wlodarczyk, Quasi-power laws in multi-particle production processes, *Chaos Soliton Fract.* (2015), DOI:10.1016/j.chaos.2015.04.016

[22] V. Garz, Stability of freely cooling granular mixtures at moderate densities, *Chaos Soliton Fract.* (2015) DOI:10.1016/j.chaos.2015.07.022

[23] F. Corberi, G. Gonnella and A. Mossa, Condensation and equilibration in an urn model, *Chaos Soliton Fract.* (2015), DOI:10.1016/j.chaos.2015.05.012

[24] R.T. Sibatov and V.V. Svetukhin, Fractional kinetics of subdiffusion-limited decomposition of a supersaturated solid solution, *Chaos Soliton Fract.* (2015), DOI:10.1016/j.chaos.2015.04.002

[25] F. Furstenberg, M. Dolgushev and A. Blumen, Exploring the applications of fractional calculus: Hierarchically built semiflexible polymers, *Chaos Soliton Fract.* (2015), DOI:10.1016/j.chaos.2015.07.006

[26] U.K. Basak, A. Datta, Fractal dimension and complexity in the long-term dynamics of a monomolecular layer, *Chaos Soliton Fract.* (2015), DOI:10.1016/j.chaos.2015.08.008

[27] P. Bhadola, N. Deo, Study of RNA Structures with a Connection to Random Matrix Theory, *Chaos Soliton Fract.* (2015), DOI: **DOI TO BE INSERTED**

[28] D. Delle Side, E. Giuffreda, S.M. Tredici, A. Talá, C. Pennetta and P. Aliano, Quorum Sensing: Complexity in the bacterial world, *Chaos Soliton Fract.* (2015), DOI:10.1016/j.chaos.2015.05.011

[29] L.F. Cugliandolo, G. Gonnella and A. Sumac, Fluctuations of rotational and translational degrees of freedom in an interacting active dumbbell system, *Chaos Soliton Fract.* (2015), DOI:10.1016/j.chaos.2015.04.015

[30] M. Patriarca, E. Hernández-García and R. Toral, Constructive effects of diversity in a multi-neuron model of the homeostatic regulation of the sleep-wake cycle, *Chaos Soliton Fract.* (2015), DOI: **DOI TO BE IN-**

SERTED

[31] P. Grigolini, Emergence of biological complexity: Criticality, renewal and memory, *Chaos Soliton Fract.* (2015), DOI:10.1016/j.chaos.2015.07.025