

Removing achromatic reflections from color images with application to artwork imaging

Luigi Bedini, Pasquale Savino, and Anna Tonazzini

Istituto di Scienza e Tecnologie dell'Informazione
Area della Ricerca CNR di Pisa
Via G. Moruzzi, 1, I-56124 PISA, Italy
Email: {name.surname}@isti.cnr.it

Abstract—We propose a general approach to remove reflections from a color image acquired through a semi-transparent medium, and show its application to the restoration of images of paintings framed behind glass and manuscripts laminated for conservation purposes, affected by the reflection of a light source. The problem is modeled by assuming that the unwanted reflection is an achromatic or monochromatic image that combines additively with the real transmitted image of the object of interest. In the absence of information about the mixing coefficients, we adopted a blind source separation technique that exploits the dependence of the three color channels of the original image, and the independence of the reflected image. In particular, these constraints are forced on the image gradients rather than on the intensity images. The algorithm is constituted of a step for the estimation, via independent component analysis of the model parameters, followed by a regularization technique to estimate the component images. The algorithm is very fast and provides promising results.

Keywords—Reflection removal, independent component analysis, image regularization, color document restoration

I. INTRODUCTION

The removal of reflections from color images acquired through a semi-transparent medium is a common need for both recreational and professional photographers. This removal is necessary both for improving the human perception and for subsequent application of many computer vision algorithms. This problem can occur also in the Cultural Heritage field. Indeed, historical artworks, such as paintings, frescos, tapestries and precious manuscripts, are often framed behind glass for preservation purposes. Furthermore, in case of manuscripts or drawings, still for conservation, in the past they might have been covered by some chemical material producing a sort of plastic coat. Taking a photograph of these artworks for digitization can cause the resulting image to appear as the superposition of the real transmitted image of the desired object and a virtual reflected image of another object or a light source located in front of it.

The earliest approaches to avoid reflection were based on polarimetric imaging [1] [2], e.g., by incorporating a polarizer into the optical system [3][4]. Cameras equipped with a liquid crystal polarizer were also proposed [5]. Nevertheless, the polarizer is capable of completely removing the reflected component only when the viewing angle is equal to the Brewster angle [6], which is hard to achieve. Furthermore, although museums, libraries and archives are nowadays often

equipped with specialized digitization setups, the availability of polarizers, and the expertise for using them, are not so frequent, at least as far as we know.

From the analysis of optical models, in [7] it has been hypothesized that the combination of transmitted image and reflected image can be approximated as a linear mixing process. Then, many works have been devoted to the use of statistical methods of independent components analysis (ICA) for blindly separating two source images when at least two mixtures, obtained for example by using linear polarizers, are available [7]. Further improvements have been obtained using sparse ICA (SPICA) [8], also capable to handle multiple source images with different motions [9][10][11]. Although these approaches can give excellent removal of reflections, they need two or more observations, whereas the problem remains highly ill-posed when only a single image is available. In [12] local features are used to handle the problem, and in [13] sparsity prior and user provided information have been exploited. Within a MAP estimation framework, color channel dependency of the transmitted image, and independence of the achromatic reflection, is proposed in [14].

Another drawback is that in all the methods above the pixels are assumed to share the same mixing coefficients, which is unlikely in most real-life scenarios. Spatially varying mixtures have been considered in [15] to separate transparent layers through layer information exchange, and in [16], where the basic Bayesian model of [14] has been augmented to incorporate information about the differences in structure of the transmitted and the reflected image.

In this paper, we retain the stationary linear mixing model of [14], and propose to overcome the highly under-determination of the problem by exploiting reasonable constraints about the coincidence of the gradients of the three color channels of the normalized transmitted image, and the statistical independence of the gradient of the normalized reflected image from those of the normalized transmitted one. However, rather than expensive MAP estimation strategies, we propose a very fast algorithm that acts in two separate and subsequent steps, both exploiting the same kind of constraints. In a first step, we adopt fast independent component analysis (ICA) algorithms to estimate the model parameters. In a second step, once the data model is fully known, we adopt regularization techniques to estimate the four component images. The method is devised for application to color images of any kind of scene affected by an achromatic or monochromatic reflection, and, in

particular, we show here its application to images of paintings and ancient manuscripts affected by a light reflection, which is typical of acquisitions made through a glass, or of photographs of plastic coated documents.

The paper is organized as follows. In Section 2, we describe the model adopted for the reflection effect, and the a priori assumptions that we exploit to reduce the indeterminacy of the estimation problem. Section 3 describes our strategy for estimating the model parameters, whereas in Section 4 the method employed to separate the reflected image from the observed image is detailed. Section 5 is devoted to the experimental results we obtained in the cases of photographs of paintings and manuscripts taken through a semi-transparent medium. Finally, Section 6 concludes the paper, by discussing some ideas that could help to overcome the present limitations of the method.

II. DATA MODEL

The data model we consider is the following [14]:

$$\begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} = \begin{bmatrix} a_{11} & 0 & 0 & a_{14} \\ 0 & a_{22} & 0 & a_{24} \\ 0 & 0 & a_{33} & a_{34} \end{bmatrix} \begin{bmatrix} s_1(t) \\ s_2(t) \\ s_3(t) \\ s_4(t) \end{bmatrix} \quad t = 1, \dots, N \quad (1)$$

where $x_1(t)$, $x_2(t)$ and $x_3(t)$, are the Red, Green and Blue color channels of the observed image at pixel t , $s_1(t)$, $s_2(t)$ and $s_3(t)$ are the ideal colors of the same pixel in the transmitted source image, $s_4(t)$ represents the achromatic reflected component, and a_{ij} are the coefficients of the transparencies of a semi-transparent medium to different light colors. By indicating with A the 3×4 matrix above, eq. (1) can be rewritten in vectorial form as $\mathbf{x} = A\mathbf{s}$, where \mathbf{x} is a $3 \times N$ matrix and \mathbf{s} is a $4 \times N$ matrix, and can be partitioned as:

$$\mathbf{x} = A_c \mathbf{s}_c + A_m \mathbf{s}_m \quad (2)$$

with

$$A_c = \begin{bmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{bmatrix}, \quad A_m = \begin{bmatrix} a_{14} & 0 & 0 \\ 0 & a_{24} & 0 \\ 0 & 0 & a_{34} \end{bmatrix} \quad (3)$$

$$\mathbf{s}_c = [s_1 \quad s_2 \quad s_3]^T, \quad \mathbf{s}_m = [s_4 \quad s_4 \quad s_4]^T \quad (4)$$

where T means transposition.

Our aim would be to estimate the ideal transmitted image \mathbf{s}_c from the data \mathbf{x} . This problem is clearly highly under-determined, since it has more unknowns than data and some parameters to be estimated. However, some reasonable assumptions, based on a priori information, can be taken to reduce its ill-posedness and define a unique solution. To this purpose, we exploit constraints on the mutual dependence of the edges of the source images, and, specifically, assume that the edges of the three color channels s_1 , s_2 and s_3 coincide, whereas they are independent of the edges of s_4 .

Let us apply a high-pass filter to the data, to derive gradient maps where the edges are highlighted, and call T_c and T_m

the two following diagonal matrices that make the high-pass filtered versions of \mathbf{s}_c and \mathbf{s}_m to have unit variance:

$$T_c^{-1} = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix}, \quad T_m^{-1} = \begin{bmatrix} \sigma_4 & 0 & 0 \\ 0 & \sigma_4 & 0 \\ 0 & 0 & \sigma_4 \end{bmatrix} \quad (5)$$

being σ_1 , σ_2 , σ_3 and σ_4 the unknown standard deviations of the high-pass filtered versions of s_1 , s_2 , s_3 and s_4 , respectively. Calling \mathbf{y} the high-pass filtered data, and \mathbf{z}_c and \mathbf{z}_m the unit variance, high-pass filtered variables, it is:

$$\mathbf{y} = A_c T_c^{-1} \mathbf{z}_c + A_m T_m^{-1} \mathbf{z}_m \quad (6)$$

where

$$A_c T_c^{-1} = \begin{bmatrix} a_{11}\sigma_1 & 0 & 0 \\ 0 & a_{22}\sigma_2 & 0 \\ 0 & 0 & a_{33}\sigma_3 \end{bmatrix} \quad (7)$$

and

$$A_m T_m^{-1} = \begin{bmatrix} a_{14}\sigma_4 & 0 & 0 \\ 0 & a_{24}\sigma_4 & 0 \\ 0 & 0 & a_{34}\sigma_4 \end{bmatrix} \quad (8)$$

In this way, we have reformulated the original data model of eq. (2) in terms of the gradients of the source images, rather than in terms of their intensities, and, in the following section, we will show that this enables finding strategies to estimate the model parameters.

III. ESTIMATION OF THE TRANSPARENCY COEFFICIENTS

Eq. (6) represents a system of 3 equations and 4 unknown. All these equations share the same total of 6 unknown coefficients of matrices $A_c T_c^{-1}$ and $A_m T_m^{-1}$. Let us consider now the covariance matrix of the filtered data, $E[\mathbf{y}\mathbf{y}^T]$, where E indicates expectation. From the hypotheses of independence of \mathbf{z}_c and \mathbf{z}_m , it is:

$$E[\mathbf{y}\mathbf{y}^T] = A_c T_c^{-1} E[\mathbf{z}_c \mathbf{z}_c^T] A_c T_c^{-1} + A_m T_m^{-1} E[\mathbf{z}_m \mathbf{z}_m^T] A_m T_m^{-1} \quad (9)$$

with

$$E[\mathbf{z}_c \mathbf{z}_c^T] = E[\mathbf{z}_m \mathbf{z}_m^T] = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \quad (10)$$

It is apparent that eq. (9), in view of eqs. (10), establishes a relationship between the 6 coefficients of the symmetric covariance matrix, which can be estimated from the data, and the 6 unknown coefficients of matrices $A_c T_c^{-1}$ and $A_m T_m^{-1}$. Thus, at least in principle, these coefficients can be estimated.

To this purpose, an obvious way is to invert eq. (9) by least mean squares and gradient descent algorithms. However, we have observed a slow convergence and not much precise estimates. Alternatively, still exploiting the substantial coincidence of the edges of the ideal image channels, that is $\mathbf{z}_1 = \mathbf{z}_2 = \mathbf{z}_3$, and the independence of \mathbf{z}_4 from \mathbf{z}_1 (as well as from \mathbf{z}_2 and \mathbf{z}_3), the 3×4 system of eq. (6) can be partitioned into the following 3 independent 2×2 systems:

$$\begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \end{bmatrix} = \begin{bmatrix} a_{11}\sigma_1 & a_{14}\sigma_4 \\ a_{22}\sigma_2 & a_{24}\sigma_4 \end{bmatrix} \begin{bmatrix} \mathbf{z}_1 \\ \mathbf{z}_4 \end{bmatrix} \quad (11)$$

$$\begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_3 \end{bmatrix} = \begin{bmatrix} a_{11}\sigma_1 & a_{14}\sigma_4 \\ a_{33}\sigma_3 & a_{34}\sigma_4 \end{bmatrix} \begin{bmatrix} \mathbf{z}_1 \\ \mathbf{z}_4 \end{bmatrix} \quad (12)$$

$$\begin{bmatrix} \mathbf{y}_2 \\ \mathbf{y}_3 \end{bmatrix} = \begin{bmatrix} a_{22}\sigma_2 & a_{24}\sigma_4 \\ a_{33}\sigma_3 & a_{34}\sigma_4 \end{bmatrix} \begin{bmatrix} \mathbf{z}_1 \\ \mathbf{z}_4 \end{bmatrix} \quad (13)$$

From the above systems, the mixing coefficients can be estimated through Independent Component Analysis (ICA) algorithms.

This strategy leaves however some indeterminacies. First of all, some coefficients are estimated twice, and slightly different values can be obtained. A good remedy in this respect is to retain the average value as the unique estimate for those coefficients. More serious problems arise instead from the well known scale and permutation indeterminacies that are intrinsic in the ICA solutions. As per the scale, we could obtain negative values for the coefficients. Again, this can be easily solvable, since we know that the coefficients must be positive, so that it suffices to invert their sign. The permutation indeterminacy means that the columns of the mixing matrices can be occasionally and unpredictably inverted. Note however that, in order to estimate all the 6 coefficients, only two of the above equations are sufficient, let us say the first one and the second one. The mixing matrices of these two equations should approximately share the same first row. Thus, a similarity check between the two estimated matrices can be used to assign the same order to their columns, the right one in both or the inverted one in both. Since we cannot discriminate the two situations, this means that we have to reconstruct two different pairs of solutions, each constituted of the supposed restored image plus its removed reflection, and then choose the correct pair by visual inspection.

The method employed to reconstruct the 4 source images for any set of estimated parameters will be described in the following section.

IV. ESTIMATION OF THE TRANSMITTED AND REFLECTED SOURCE IMAGES

On the basis of the transparency coefficients estimated (up to the standard deviations) with the ICA method described above, the data system of eq. (2) can be rewritten as:

$$\begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \mathbf{x}_3 \end{bmatrix} = \begin{bmatrix} a_{11}\sigma_1 & 0 & 0 & a_{14}\sigma_4 \\ 0 & a_{22}\sigma_2 & 0 & a_{24}\sigma_4 \\ 0 & 0 & a_{33}\sigma_3 & a_{34}\sigma_4 \end{bmatrix} \begin{bmatrix} \mathbf{s}_1/\sigma_1 \\ \mathbf{s}_2/\sigma_2 \\ \mathbf{s}_3/\sigma_3 \\ \mathbf{s}_4/\sigma_4 \end{bmatrix} \quad (14)$$

or, equivalently:

$$\begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \mathbf{x}_3 \end{bmatrix} = \begin{bmatrix} a_{11}\sigma_1 & 0 & 0 \\ 0 & a_{22}\sigma_2 & 0 \\ 0 & 0 & a_{33}\sigma_3 \end{bmatrix} \begin{bmatrix} \mathbf{s}_1/\sigma_1 \\ \mathbf{s}_2/\sigma_2 \\ \mathbf{s}_3/\sigma_3 \end{bmatrix} + \begin{bmatrix} a_{14}\sigma_4 \\ a_{24}\sigma_4 \\ a_{34}\sigma_4 \end{bmatrix} \mathbf{s}_4/\sigma_4 \quad (15)$$

This is a linear system with 3 equations and 4 unknowns, i.e., \mathbf{s}_1/σ_1 , \mathbf{s}_2/σ_2 , \mathbf{s}_3/σ_3 , and \mathbf{s}_4/σ_4 , then, again, an ill-posed

problem. Assuming to be able to find a solution, it is:

$$\begin{aligned} A_c \mathbf{s}_c &= \begin{bmatrix} a_{11}\sigma_1 & 0 & 0 \\ 0 & a_{22}\sigma_2 & 0 \\ 0 & 0 & a_{33}\sigma_3 \end{bmatrix} \begin{bmatrix} \mathbf{s}_1/\sigma_1 \\ \mathbf{s}_2/\sigma_2 \\ \mathbf{s}_3/\sigma_3 \end{bmatrix} = \\ &= \mathbf{x} - \begin{bmatrix} a_{14}\sigma_4 \\ a_{24}\sigma_4 \\ a_{34}\sigma_4 \end{bmatrix} \mathbf{s}_4/\sigma_4 \end{aligned} \quad (16)$$

Note that it is impossible to disentangle A_c and \mathbf{s}_c , unless extra information on the scale of \mathbf{s}_c is available. Thus, accepting this ambiguity, we can only estimate the image free of reflection, $A_c \mathbf{s}_c$, rather than the true transmitted image \mathbf{s}_c . The price to be paid is that we cannot expect a full fidelity of the colors of the reconstructed image. However, when the application is in the document analysis field, this may be a minor problem, since the essential aspect is the possibility to recover the legibility of the text, often compromised by the presence of the reflection that masks it.

To estimate the 4 source images, we solve the system of eq. (14) by constrained least squares, where the constraints enforce the minimum distance between the three pairs $(\mathbf{s}_1/\sigma_1, \mathbf{s}_2/\sigma_2)$, $(\mathbf{s}_1/\sigma_1, \mathbf{s}_3/\sigma_3)$ and $(\mathbf{s}_2/\sigma_2, \mathbf{s}_3/\sigma_3)$ of the color channels of the transmitted image, and the maximum orthogonality between each of such channels and the achromatic reflected image \mathbf{s}_4/σ_4 . To this purpose, we define an energy function to be minimized with respect to \mathbf{s}_i/σ_i , $i = 1, 2, 3, 4$, which is constituted by a data term, i.e., the squared Euclidean norm of the difference between the left hand side and the right hand side of eq. (14), plus two quadratic stabilizer terms expressing the two constraints above, weighted by two positive regularization parameters λ_1 and λ_2 , respectively. Calling A_σ the mixing matrix in eq. (14) and \mathbf{s}_σ the vector of the four unknown component images, the energy function is:

$$\phi(\mathbf{s}_\sigma) = \|\mathbf{x} - A_\sigma \mathbf{s}_\sigma\|^2 + \lambda_1 \mathbf{s}_\sigma^T S_1 \mathbf{s}_\sigma + \lambda_2 \mathbf{s}_\sigma^T S_2 \mathbf{s}_\sigma \quad (17)$$

where

$$S_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix}, \quad S_2 = \begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 2 & -1 & 0 \\ -1 & -1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (18)$$

Being the whole energy function quadratic, the solution can be obtained in close form. This solution depends however on λ_1 and λ_2 , that is we will obtain $\mathbf{s}_i(\lambda_1, \lambda_2)/\sigma_i$, $i = 1, 2, 3, 4$. The minimization of the energy function is then iteratively repeated for λ_1 and λ_2 in a suitable range, until the following condition is satisfied at best:

$$E[\mathbf{z}(\lambda_1, \lambda_2) \mathbf{z}(\lambda_1, \lambda_2)^T] = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (19)$$

where

$$\mathbf{z}(\lambda_1, \lambda_2) = [\mathbf{z}_1(\lambda_1, \lambda_2) \quad \mathbf{z}_2(\lambda_1, \lambda_2) \quad \mathbf{z}_3(\lambda_1, \lambda_2) \quad \mathbf{z}_4(\lambda_1, \lambda_2)]^T \quad (20)$$

It is worth noting that the search for the optimal regularization parameters is the most expensive part of the entire algorithm, which is however very fast.

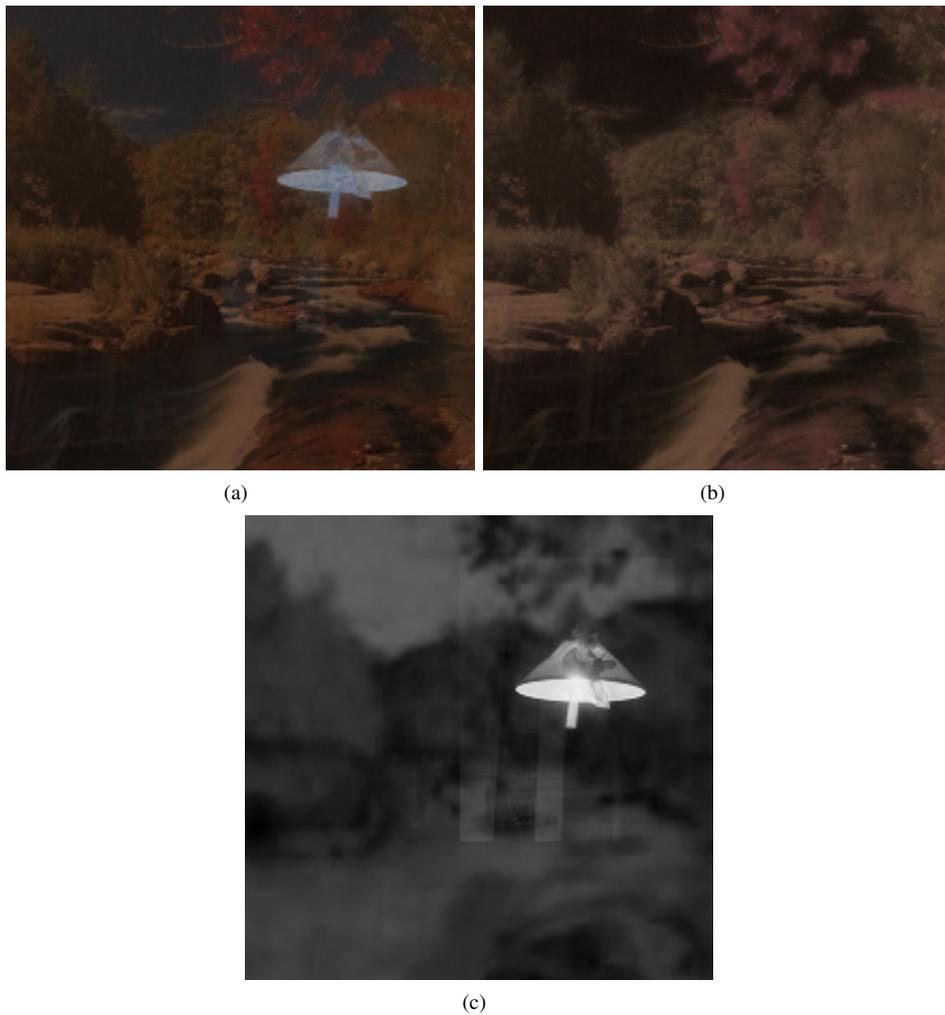


Fig. 1. Removal of reflection from the picture of a painting: (a) original acquired image; (b) restored image; (c) separated reflection.

V. EXPERIMENTAL RESULTS

Our algorithm has been designed for the general problem of removing reflection from images of any type. We show here its application to images of artworks. We start with a picture of a painting framed by a glass. Figure 1(a) shows the acquired image where the reflection of a lamp is well visible. Although separation of the two images has not been completely achieved, as it can be noticed from some residual patterns belonging to the transmitted image left in the reflected image of Figure 1(c), Figure 1(b) shows the transmitted image perfectly freed from reflection. However, due to the imperfect separation, together with the above mentioned scale ambiguity left in the solution, the colors are not perfectly reproduced.

In a subsequent experiment, we show the results obtained with this algorithm when the reflection affects a document image. Figure 2(a) shows the image of a plastic coated ancient manuscript. It is apparent that the light reflection prevent reading part of the text. With our algorithm we have been able to separate the reflected image (Figure 2(c)), and recover the free of reflection image of Figure 2(b). Note that the text under the reflection is now well visible.

VI. CONCLUSIONS

We proposed a method to remove a reflection from color images acquired through a semi-transparent medium. The method assumes that the unwanted reflection, often caused by a light source, is an achromatic image that combine additively with the real transmitted image of the object of interest. Since the mixing coefficients are unknown, we adopt a blind source separation technique exploiting the substantial coincidence of the gradients of the three color channels of the original image, and the independence of the gradient of the reflected image. The algorithm acts in two step. In the first one, the model parameters are estimated through ICA, whereas, in the second step, the four component images are estimated via regularization. We presented the promising results of the application of this algorithm to the restoration of both real scenes acquired through a glass and manuscripts laminated for conservation purposes. Nevertheless, some aspects of the methods still need to be investigated and improved, especially with respect to the step of estimation of the component images. These issues regards the search for an efficient way to overcome the permutation indeterminacy of ICA and thus determine the correct order of the transparency coefficients, and the look for more effective constraints about the component images,

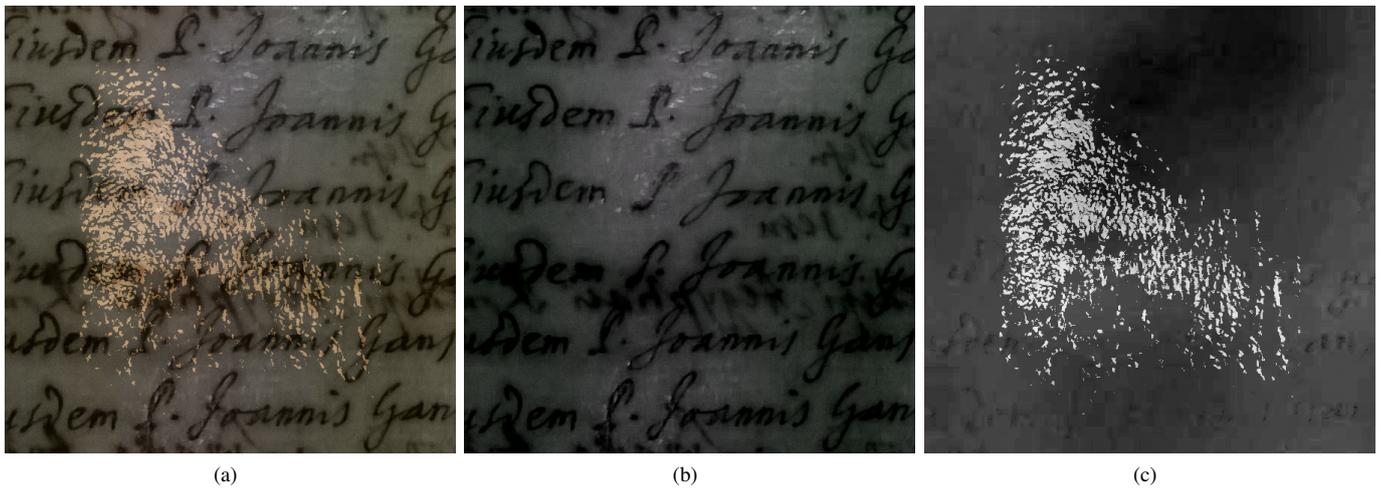


Fig. 2. Removal of reflection from the picture of an ancient manuscript: (a) original acquired image; (b) restored image; (c) separated reflection.

in order to be able to estimate their scale and then, by discriminating A_c from s_c , obtain a more faithful color of the reconstructed images.

VII. ACKNOWLEDGEMENTS

This work has been supported by European funds, through the program POR Calabria FESR 2007-2013 - PIA Regione Calabria Pacchetti Integrati di Agevolazione Industria Artigianato Servizi, project ITACA (Innovative Tools for cultural heritage ArChiving and restorAtion).

REFERENCES

- [1] T. Cronin, N. Shashar, and L. Wolff, "Portable imaging polarimeters," in *Proc. ICPR 1994*, vol. A, 1994, pp. 606–609.
- [2] K. Nayar, X. Fang, and T. Boult, "Separation of reflection components using color and polarization," *Int. J. Comput. Vis.*, vol. 21, pp. 163–186, 1997.
- [3] J. S. Y.Y. Schechner and N. Kiryati, "Polarization-based decorrelation of transparent layers: the inclination angle of an invisible surface," in *Proc. Int. Conf. on Computer Vision 1999*, 1999, pp. 814–819.
- [4] Y. Schechner, J. Shamir, and N. Kiryati, "Polarization and statistical analysis of scenes containing a semireflector," *Journal of the Optical Society of America A*, vol. 17, pp. 276–284, 2000.
- [5] H. Fujikake, K. Takizawa, T. Aida, H. Kikuchi, T. Fujii, and M. Kawakita, "Electrically-controllable liquid crystal polarizing filter for eliminating reflected light," *Opt Rev*, vol. 5, pp. 93–98, 1998.
- [6] M. Born and E. Wolf, *Principles of Optics*. London: Pergamon, 1965.
- [7] H. Farid and E. Adelson, "Separating reflections and lighting using independent components analysis," in *Proc. CVPR 1999*, vol. 1, 1999, pp. 262–267.
- [8] A. Bronstein, M. Bronstein, M. Zibulevsky, and Y. Zeevi, "Sparse ICA for blind separation of transmitted and reflected images," *Int. Journal of Imaging System and Technology*, vol. 15, pp. 84–91, 2005.
- [9] E. Beery and A. Yeredor, "Blind separation of reflections with relative spatial shifts," in *Proc. ICASSP 2006*, vol. 5, 2006, pp. 625–628.
- [10] K. Gai, Z. Shi, and C. Zhang, "Blindly separating mixtures of multiple layers with spatial shifts," in *Proc. CVPR 2008*, 2008, pp. 1–8.
- [11] —, "Blind separation of superimposed images with unknown motions," in *Proc. CVPR 2009*, 2009, pp. 1881–1888.
- [12] A. Levin, A. Zomet, and Y. Weiss, "Separating reflections from a single image using local features," in *Proc. ECCV 2004*, 2004, pp. 306–313.
- [13] A. Levin and Y. Weiss, "User assisted separation of reflections from a single image using a sparsity prior," *IEEE Trans. Pattern Analysis and Machine Intelligence*, vol. 29, pp. 1647–1655, 2007.
- [14] K. Kayabol, E. Kuruoglu, and B. Sankur, "Image source separation using color channel dependencies," in *Proc. 8th Int. Conf. on Independent Component Analysis and Signal Separation*, 2009, pp. 499–506.
- [15] B. Sarel and M. Irani, "Separating transparent layers through layer information exchange," in *Proc. ECCV 2004*, ser. LNCS, T. Pajdla and J. Matas, Eds., vol. LNCS 3024. Springer, Heidelberg, 2004, pp. 328–341.
- [16] Q. Yan, E. E. Kuruoglu, X. Yang, Y. Xu, and K. Kayabol, "Separating reflections from a single image using spatial smoothness and structure information," in *Proc. LVA/ICA 2010*, ser. Lecture Notes in Computer Science, V. V. et al., Ed., vol. LNCS 6365. Springer, 2010, pp. 637–644.