LARASE: Testing General Relativity with Satellite Laser Ranging

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LARASE is an experiment devoted to test General Relativity in its weak-field linearized approximation using the geodetic satellites LAGEOS and LARES. One main target is the measurements of the Lense-Thirring effect with an accuracy higher than in the past. We present the LARASE activitities and a preliminary measurement of the effect.

1 Introduction

The Solar System is the classical laboratory for testing General Relativity (GR) in its weak field and slow motion (WFSM) limit. Measurement of the so-called gravitomagnetic effects represents one of the classical goals in this field. In GR, rotating bodies drag a freely falling reference frame, causing a precession that takes the name of Lense-Thirring [1](L-T) or gravitomagnetic effect. "Mass-currents" produce effects on the near moving masses, a phenomenon that is formally equivalent to magnetism. Einstein named this effect frame-dragging.

A first accurate measurement of the L-T effect was obtained by Ciufolini and Pavlis [2] in 2004. They reached an uncertainty of 10% in the evaluation of the main systematic error sources by analyzing about 11 years of the Satellite Laser Ranging measurements of the two LAGEOS satellites. The NASA/Stanford experiment Gravity Probe B (GP-B)[3] in 2011 succeeded in the measurement of frame dragging by measuring the precession of superconducting gyroscopes orbiting the Earth, the so-called Schiff effect. The uncertainty was about 20%.

The main scientific goal of the LAser RAnged Satellites Experiment (LARASE) is the measurement of the Lense-Thirring effect using the data of the two LAGEOS and of the newer LARES satellite, with improved precision and accuracy with respect to the best measurements to date. We refer to Lucchesi et al. [4] for a detailed description of the program.

2 The satellites of the LARASE experiment

The "tools" of LARASE measurements are geodetic satellites that orbit around the Earth. They are equipped with corner cube reflectors (CCR) and tracked with great precision using laser beams from a network of stations of the International Laser Ranging Service (ILRS) [5]. The ILRS stations provide satellites' range data, obtained with the measurement of the round-trip time, that have currently reached RMS errors around one millimeter: this allows reconstruction of the orbit, using precise orbit determination (POD) techniques, at a level of about 1 *cm*. The

ILRS data are freely available, both as high frequency measurements (called full rate) and as "normal points", i.e. values averaged over a period of two minutes for the LAGEOS satellites. We use ranging data for the two LAGEOS and the LARES satellites. The first LAGEOS was launched in 1976 by NASA, has a mass of about 400 *kg*, a diameter of 60 *cm* and hosts 426 CCR. LAGEOS II, launched in 1992 by NASA/ASI, is almost identical to LAGEOS.

LARES was launched in 2012 by ASI, it is smaller but denser than LAGEOS, with a diameter of 36 *cm* and a mass of about 390 *kg*. It is equipped with 92 CCR. The mean orbital parameters of the three satellites are reported in table 1. One relevant difference between LAGEOS and LARES is the height of their orbit: indeed, LARES is on a much lower orbit, and is therefore more sensitive to the non-uniformity of the Earth's gravitational field (higher multipoles), to tidal effects and to the drag effects from the residual atmosphere.

Table 1: Keplerian parameters of the LAGEOS and LARES satellites. The right ascension of the ascending node and argument of the pericenter are estimated at October 24th, 1992 for LAGEOS and LAGEOS II, and at February 18th, 2012 for LARES.

Orbital Element		LAGEOS	LAGEOS II	LARES
Semi-major axis (m)	a	$1.23 \cdot 10^7$	$1.22 \cdot 10^7$	$7.82 \cdot 10^6$
Eccentricity	e	$4.43 \cdot 10^{-3}$	$1.38 \cdot 10^{-2}$	$1.20 \cdot 10^{-3}$
Inclination (deg.)	i	109.84	52.66	69.49
Right ascension of the ascending node (deg.)	Ω	289.74	113.75	230.84
Argument of pericenter (deg.)	ω	53.12	212.57	296.99

3 LARASE activities

The main objective of LARASE is the measurement of the L-T effect with an improved error budget: to this purpose, we need to account for all perturbations and to keep under tight control all the systematic errors, in order to produce a robust, reliable and uncontestable result. This target requires a careful study of all the effects of both gravitational and non-gravitational origin that act on the satellite and may mask the GR effects.

The main observables used for the measurement are the rate of right ascension of the ascending node $\dot{\Omega}$ of the orbit of the satellites. Also the rate of the argument of pericenter $\dot{\omega}$ of the orbit can be used. Both these drifts are proportional to the gravitomagnetic parameter μ, which is equal to 1 in Einstein's GR and 0 in Newtonian classical physics:

$$\dot{\Omega}^{LT} = \mu \frac{2GJ_\oplus}{c^2 a^3 \left(1-e^2\right)^{\frac{3}{2}}} \qquad \dot{\omega}^{LT} = \mu \frac{-6GJ_\oplus}{c^2 a^3 \left(1-e^2\right)^{\frac{3}{2}}} \cos i \tag{1}$$

where G is the gravitational constant, a the semi-major axis, e the eccentricity, i the inclination, c the speed of light and finally $J_\oplus$ is the Earth's angular momentum.

Other phenomena, beside GR, can cause the drift of the satellite node Ω and pericenter ω. These parameters are perturbed also in the Newtonian sense: indeed, a similar effect can be produced by the non-sphericity of Earth's mass distribution, and in particular by the even terms of the geopotential multipole expansion. This disturbing effect can be reduced using the measured Keplerian parameters of several satellites and properly combining their residuals [7,8]. For example, we can use a combination of the right ascension of the nodes of the 3 satellites [8]:

$$\dot{\Omega}_{comb} = \dot{\Omega}_{L1} + k_1 \dot{\Omega}_{L2} + k_2 \dot{\Omega}_{LR}, \tag{2}$$

This weighted sum, where the coefficients k_1 and k_2 are defined by the orbital parameters, defines an equivalent shift $\dot{\Omega}_{comb}$ that is insensitive to all errors related to the first two even zonal harmonics, i.e. the quadrupole and octupole coefficients of the gravitational field. Alternative combinations are possible using also the argument of pericenter of LAGEOS II, the most eccentric among the 3 satellites [9].

In order to reduce and properly evaluate the errors affecting the relativistic measurements, a careful study of the gravitational and non-gravitational forces acting on the satellite is required. The recent activities of LARASE have focused on:

- Study of thermal effects: these have an impact on the satellite attitude dynamics. We started with a critical analysis of the satellites physical characteristics available in literature: this enabled us to build 3D models, to evaluate e.g. the moments of inertia [6] of the satellites. These models will also be used for thermal studies based on finite-element techniques. Figure 1 shows a rendering of LARES obtained using the 3D model of the satellite.

- Study of neutral particle drag effects: due to its lower orbit, LARES is more sensitive to the perturbing effects of the neutral drag. Using a modified version of SATRAP [10] software, we evaluated the perturbations acting on LARES, in particular the decay of its semi-major axis [11]. We considered the evolution of solar and geomagnetic activities using different thermospheric density models.

- Study of Earth tides, both solid and ocean: the tides of the Earth affect the dynamics of the satellites in a threefold way: i) they modify the Earth's shape, changing the position of the ground stations, ii) they change the Earth gravitational field that influences the orbit of the satellites and iii) they change the reference frame adopted in the measurements, as they modify the Earth's rotation. Also in this case a larger effect is expected on LARES with respect to the two LAGEOS.

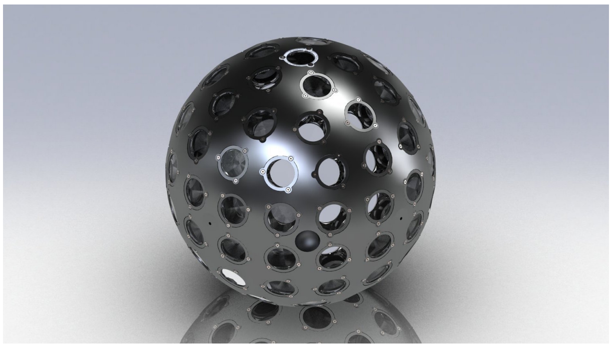

Figure 1 – CAD Rendering of LARES satellite

4 Preliminary results

We show a preliminary analysis on the ranging data of about 3.4 years of the LAGEOS and LARES satellites. The data were subdivided in single time spans (*arcs*) of 7 days. The *best orbit* was estimated on each arc using a least-squares approach. The measured position and velocity in each arc were used as initial conditions in a numerical evaluation of the orbit based on a model that includes all known effects, except the L-T effect to be measured. The reconstructed (propagated) orbit at the end of each arc and the measured orbit at the same initial epoch of the subsequent arc are compared, obtaining the so called (O-C) or *residuals*. These residuals should contain the "displacement" due to the L-T precession of the orbit.

The node residuals calculated for each of satellite are combined using Eq. (2). Figure 2 displays the time dependence of the integrated residuals for Ω_{comb}. Since the L-T effect produces a secular effect (see Eq. (1)), we expect a linear behaviour for the integrated residuals of the satellites nodes.

In practice, some periodic disturbances superimpose the linear behaviour predicted by GR. When these effects (typically tides and thermal effects) have periods longer than the analyzed time span, they are hard to be discriminated: on time intervals shorter than half their period, their time evolution can be confused with a monotonic trend. We fitted many of these oscillating disturbances, taking into account a number of tides (both solid and ocean) varying between 3 and 12. In this way their oscillating behaviour can be removed, at least partially.

The computed linear fit to the data is the red line in figure 2, corresponding to a drift of 50.11 $mas/year$, to be compared with the GR prediction represented by the green line, with a slope of 50.18 $mas/year$. The statistical error is therefore around 0.1%, but the sensitivity to the number of the tides absorbed in the fit can increase this error to a few percent level. A careful analysis of the main systematic error sources is underway.

We plan, for the near future, to reduce the errors that depend on the dynamical models included in the POD, especially for the thermal and gravitational effects.

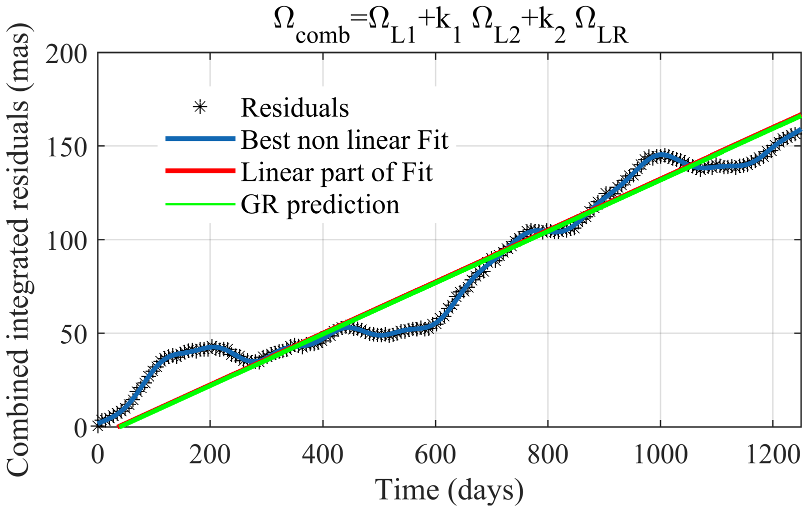

Figure 2 – Integrated residuals for the combined right ascension of the ascending node of the satellites (Ω_{comb} of eq.2). The residuals are fitted with a linear term plus a number (up to 12) of periodic terms to model slow tides.

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