An Unprecedented Empirical Rule for the Baryon and Meson Mass Spectra

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Abstract

An empirical rule, previously proposed for ground-state baryons and the only meson K, is widely extended to mesons. The rule maps pairs of hadrons (h_1, h_2), or single hadrons h_1, onto single higher-lying hadrons H. The mapping law requires that if a certain partial sum of a given power series equals the sum of the masses of hadrons h, then the whole series converges to the mass of a hadron H.

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1. - Introduction.

A belief in the simplicity of nature has generally been the leading motivation behind any search for empirical laws fitting experimental data. This was, for example, the case of the law for the wavelengths of the spectral lines of hydrogen which Balmer at first found on the basis of only four coincidences. However, for certain other aspects of the physical world, as in the case of the hadron mass spectrum, which will be dealt with here, the current point of view is rather far from reflecting such an assumption of simplicity. Nevertheless, we have applied this old "prejudice" to the problem of the hadron masses: we had wondered whether the masses of ground-state hadrons with flavored constituents might be involved in unexpectedly simple relationships. Recently, indeed, we showed that it is possible to define an empirical rule which relates the mass of certain hadrons to the sum of the masses of lighter particles.(1) These hadrons were all the strange baryons of the $J^P = 1/2^+, 3/2^+$ multiplets, the charmed baryon Λ_c, and, as the only meson considered, the strange meson K.

The main interest in this rule arose from the fact that it was possible, in particular, "to predict" the mass of the lowest-lying strange baryon, strange meson, and charmed baryon by means of the masses of lighter nonstrange or noncharmed hadrons, correspondingly. This might imply an underlying relationship between the "masses" of the different-flavor quarks, or, anyhow, between those properties of these quarks which determine the mass of a hadron.

Here, we will show that the validity of the rule can be widely extended to mesons. In particular, the evidence will be completed that this empirical rule can actually predict the mass of baryons and mesons with quarks of a certain flavor - whether strange, charmed, or bottom - using as input the masses of lighter hadrons which do not contain that particular quark. To make the present extension of the rule possible, we will give a new formulation of it, without, however, contrasting with its original presentation.

2. - The rule rewritten: invention or discovery?

We will define a correspondence which maps pairs of ground-state hadrons, (h_1, h_2), or single hadrons, h_1, onto single flavored-content hadrons, H; that is

$$(h_1, h_2) \Rightarrow H \tag{1}$$

where, in certain cases, it may be that $h_2 = 0$.

The mapping law makes use of the power-series expansion of the following function, involving hadron masses m(·),

$$m(H) = \frac{m(\mathcal{H}_0)}{\sqrt{1 - \beta}} \quad , \tag{2}$$

i.e.

$$m(H) = m(\mathcal{H}_0) \cdot \left(1 + \frac{1}{2}\beta + \frac{3}{8}\beta^2 + \frac{5}{16}\beta^3 + \dots\right) \quad , \tag{3}$$

with $0 < \beta < 1$, where $\mathcal{H}_0$ is a reference hadron with, accordingly, $m(\mathcal{H}_0) < m(H)$. Correspondence (1) is established through

$$\mathcal{S}_{\aleph}\left[\beta(H)\right] \approx \sum_i m(h_i) \tag{4}$$

with i = 1, 2, where $\mathcal{S}_{\aleph}(\beta)$ is the $\aleph$th partial sum of series (3), i.e. the sum of its first $\aleph$ terms. Integer $\aleph$ is given by

$$\aleph = n_{min}(h_1, h_2) + n(H) \quad , \tag{5}$$

where n_{min} is the minimum integer for which

$$\mathcal{S}_{(n_{min}+1)}(1) > \sum_i m(h_i) \, , \tag{6}$$

and n is determined by

$$n(H) = \text{No. of flavored constituents of H.} \quad (7)$$

We note, for example, that for mono-flavored hadrons integer n is therefore given by the absolute value of their strangeness, or charm, or bottomness. In particular, for meson $\eta'(958)$, with its strange content - namely a strange quark-antiquark pair - amounting to a fraction (probably about 50%) of its total quark content, we will consistently take $n = 1$. For meson $\phi(1020)$, which is almost a pure state $s\bar{s}$, it will be $n = 2$.

The domain of our correspondence is defined as follows:

i) hadrons h are ground-state hadrons, precisely the lowest-lying (unflavored) hadrons of the four ground-state multiplets or their flavored-content members with

$$n(h) = 1; \quad (8)$$

ii) pairs (h_1, h_2) will have total baryon number 0 or 1;

iii) for the mesons in a pair, if any, it must be that at least one belongs to the $J^{PC} = 0^{-+}$ octet and that their total strangeness and isospin satisfy

$$\sum_{\text{mesons}} S(h) = 0 \quad \text{and} \quad \sum_{\text{mesons}} I(h) \neq 0\,; \quad (9)$$

iv) if $h_2 = 0$, then h_1 will be flavored with spin $J(h_1)$ minimum;

v) finally, our pairs of hadrons h will have total mass in the interval $[m(\mathcal{H}_0), 2.62 \cdot m(\mathcal{H}_0)]$ (this excludes cases other than the only one which will be considered if $\mathcal{H}_0 = \pi^0$).

It will result that hadrons h_1, h_2, and H obey the following conditions:

I) the two sides of eq.(1) will conserve the baryon number;

II) for the values of spin of our hadrons: $J(h_i) \leq \max \left\{ J(H), J(\mathcal{H}_0) \right\}$;

III) specifically, for mono-flavored hadrons H, if no corresponding hadron h contains the same flavor, then $h_i = \pi$ for at least one value of i;

IV) finally, when H is a meson, $n(H) = 2$ if and only if $h_i \neq \pi$ for all i.

Let us now check correspondence (1). Write two sequences of permitted pairs of ground-state hadrons (h_1, h_2) (with $h_2 = 0$, if it is the case), each of the two sequences being ordered separately by increasing values of $\sum m(h_i)$, with the total baryon number equal to 1, for one sequence, and 0, for the other (see table 1). The reference hadron, $\mathcal{H}_0$, is the proton (p) for the sequence with total baryon number 1, and $\pi^0(135)$ or $\rho(770)$ for the other sequence with zero baryon number. We find, indeed, that for every pair (h_1, h_2) of the two sequences there exists a permitted, higher-lying hadron H with the right values of mass and n so as to satisfy eq.(4) with a good approximation.

These hadrons H contain flavored constituents and are, except that in one only case, ground-state hadrons. The values of mass of hadrons H, m(H), reported in table 1 are calculated from the experimental masses of the corresponding hadrons h_i, by taking the average mass value of their isospin-multiplet members which are involved in the various charge-conserving variants of a same instance of correspondence (1). The experimental uncertainties of the masses entering the calculations produce the reported uncertainties for the values of m(H) (only those ≥1 MeV are given). The experimental masses are taken from reference[2]; the value of mass used for $\rho(770)$, as $\mathcal{H}_0$, is 766.68 MeV (which is very close to its experimental average).

On the whole, those hadrons involved in correspondence (1) as either h or H are: the fundamental hadrons of the four ground-state SU(3) multiplets, i.e. the N, $\Delta(1232)$, π and $\rho(770)$; all the strange-content hadrons of these multiplets; the lowest-lying ground-state charmed and bottom (J = 1/2) baryons and (J = 0) mesons; the ground-state J = 1 charmed and charmed-strange mesons and J = 0,1 $c\bar{c}$ mesons; and, finally, the lowest-lying χ ($c\bar{c}$) meson, with $J^{PC} = 0^{++}$. As regards the first presentation of the rule, the number of coincidences with data has thus been doubled. It is the use of $\rho(770)$ as $\mathcal{H}_0$ which makes this extension of the rule to a new sequence of mesons possible.

3. - Concluding comments.

We have shown that it is possible to define an empirical rule which maps pairs of ground-state hadrons (h_1, h_2) or single hadrons h_1 onto single, mostly ground-state hadrons H. Our mapping law requires that if a certain partial sum of a given power series equals the sum of the masses of certain hadrons h, then the whole series converges to the value of mass of a heavier hadron H. The number of terms of this partial sum of the series is determined both by the content of flavored constituents of hadron H and by which hadrons h are involved in that particular correspondence. The rule can climb the hadron mass spectrum following the criterion that the positions of suitable hadrons h in the spectrum determine the position of a higher-lying hadron H.

The high number of coincidences supporting this empirical evidence appears to be a sound guarantee against accidentality. Let us note that an eventual physical significance of these coincidences would imply unexpected consequences. In particular, as already mentioned, the capability of the rule of predicting the masses of mesons and baryons with strange, charmed, or bottom quarks using as input the masses of hadrons which do not contain that kind of quark may suggest the existence of an underlying relationship between those properties of the different-flavor quarks which are responsible for the different masses of the hadrons they form. However, the apparent implications of the rule would be even more intriguing because it puts both hadrons with different and those with the same kinds of flavored quarks into the same relationship. Although we present our series of coincidences as an empirical rule, i.e. with no theoretical justification, nevertheless we ascribe a potential physical sense to it, thus neglecting the possibility that chance may have produced such a high degree of order from data. On the other hand, we know that we are thus following a well-tested and productive way of thinking in physics.

References

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(2) Particle Data Group: Phys. Lett. **B**, **239**, 1 (1990)

Table Caption

Table 1. "Prediction" of the mass values of hadrons H, m(H), from the masses of pairs of lighter ground-state hadrons (h_1, h_2) or of single hadrons h_1.

$h_1\ h_2$	H	$\mathcal{H}_0$	n_{min}	n(H)	ℵ	m(H) (MeV)	m_{exp} (MeV)
N π	Λ	p	1	1	2	1117	1116
Λ –	Σ	"	1	1	2	1190	1193
Σ –	$\Sigma(1385)$	"	1	1	2	1388	1385
$\Lambda\,\pi$	Ξ	"	1	2	3	1314	1318
$\Sigma\,\pi$	Λ_c	"	1	1	2	2284 ± 1	2285
$\Delta(1232)\,\pi$	$\Xi(1530)$	"	1	2	3	1533 ± 4	1533
$\Sigma(1385)\,\pi$	Ω	"	1	3	4	1679 ± 2	1672
Λ_c –	Ξ_{cc}	"	4	2	6	3680 ± 10	–
$\Lambda_c\pi$	Λ_b	"	5	1	6	5594 ± 30	~5600
$\pi\,\pi$	K	π^0	3	1	4	498	496
$\rho(770)\,\pi$	$\eta'(958)$	$\rho(770)$	1	1	2	957	958
K $\bar{K}$	$\phi(1020)$	"	1	2	3	1025	1019
$\eta'(958)\,\pi$	$D^*(2010)$	"	1	1	2	2019 ± 4	2009
$\rho(770)\,\eta$	$D_s^*(2110)$	"	1	2	3	2105 ± 6	2110
$K^*(892)\ \bar{K}$	$\chi_{c0}(3415)$	"	1	2	3	3403 ± 10	3415
D –	$\eta_c(2980)$	"	4	2	6	2985 ± 4	2980
N $\bar{N}$	$J/\psi(3097)$	"	4	2	6	3093	3097
D π	B	"	5	1	6	5275 ± 17	5279

TABLE 1