

# ***Methodology used at ISTI/CNR to Assess the Sever Probability of Space Tethers***

**Carmen Pardini and Luciano Anselmo**

Space Flight Dynamics Laboratory  
ISTI/CNR, Via G. Moruzzi 1  
56124 Pisa, ITALY

**IADC WG2 AI 19.1  
“Benefits and Risks of Using Electrodynamic Tethers  
to De-orbit Spacecraft”**

**22nd IADC Meeting, 19-22 April 2004  
Abano Terme, ITALY**



# Purpose

---

The purpose of this presentation is to show the methodology used at ISTI/CNR

- To compute the fatal impact rate of meteoroids and orbital debris on space tethers;
- To assess the survivability concerns of a specific electrodynamic tether system during a de-orbiting mission,

taking into account specifications and assumptions of the IADC WG2 AI 19.1 Study Plan on  
**“Benefits and Risks of using Electrodynamic Tethers to De-orbit Spacecraft”**

# Space Debris Flux Models

---

The following space debris environment models were used to compute the debris flux on space tethers:

## Meteoroid Environment

### ➤ NASA DAS 1.5.3 Model

- Meteoroid environment taken from the Space Station Program Natural and Induced Environment Definition for Design (**SSP 30425**)

## Orbital Debris Environment

### ➤ NASA DAS 1.5.3 Model – Orbital Debris smaller than 1 mm

- Orbital debris environment taken from the NASA Orbital Debris Engineering Model (**ORDEM96**)

### ➤ ISTI/CNR CODRM-99R Model – Orbital Debris larger than 1 mm

# Tether Design

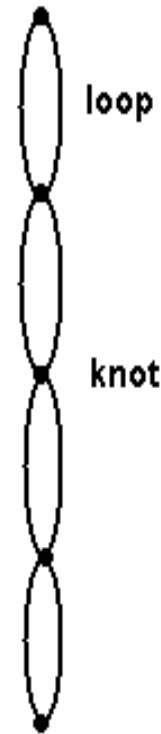
Stable cylindrical space tether deployed along the gravity gradient in a circular orbit

- **single line tether:** a single line or an envelope of lines, like the Hoytether device
- **double line tether:** the two cables are separated each other by a distance significantly larger than each line diameter and form  $n$  loops, tied together in  $n + 1$  equidistant knots along the tether

Single Wire Solution



Double Wire Solution



# Tether Cross Sectional Area

The tether is severed by a space debris provided the debris diameter ( $d$ ) satisfies the following

$$d \geq 0.25 D_T$$

and the debris edge passes within  $0.35 D_T$  of the tether centre of axial symmetry

$$D_{TC} = 2(0.35 D_T)$$

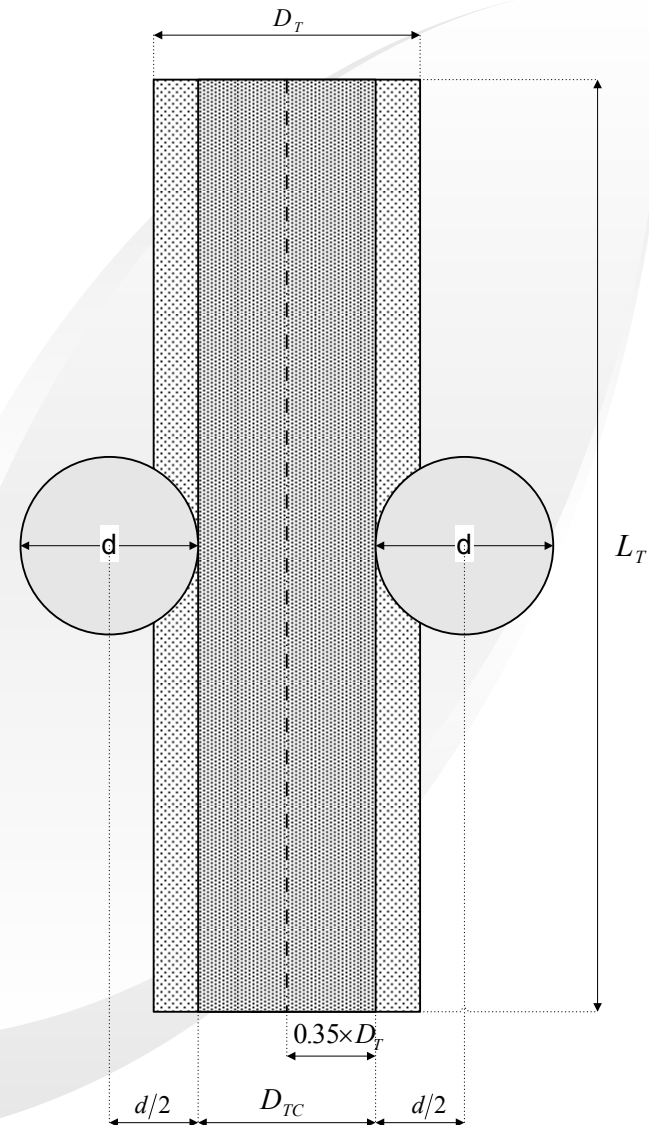
$D_{TC}$  critical tether diameter

$D_T$  tether diameter

The tether effective cross sectional area  $A_T$  is computed as:

$$A_T = L_T (D_{TC} + d)$$

with  $L_T$  tether length



# AI 19.1 Test #1

## Fatal Impact Rate Analysis

### Proposed Orbits & Tether Diameters

Orbit Altitudes [km]	Orbit Inclinations [deg]	Tether Diameter
1400	25, 50, 75	0.50 mm, 0.75 mm, 1 mm, 2.5 mm, 5 mm, 1 cm, 2.5 cm, 5 cm
1000	25, 50, 75	0.50 mm, 0.75 mm, 1 mm, 2.5 mm, 5 mm, 1 cm, 2.5 cm, 5 cm
800	25, 50, 75	0.50 mm, 0.75 mm, 1 mm, 2.5 mm, 5 mm, 1 cm, 2.5 cm, 5 cm

Tether Diameter							
0.50 mm	0.75 mm	1.00 mm	2.50 mm	5.00 mm	1.00 cm	2.50 cm	5.00 cm
Minimum Fatal Debris Diameter [mm]							
0.125	0.1875	0.25	0.625	1.25	2.5	6.25	12.5
Critical Tether Diameter [mm]							
0.35	0.525	0.7	1.75	3.5	7	17.5	35

# AI 19.1 Test #1

## Fatal Impact Rate Analysis

### Tether Diameter between 0.5 mm and 1 mm

Tether Diameter		
0.50 mm	0.75 mm	1.00 mm
Minimum Fatal Debris Diameter [mm]		
0.125	0.1875	0.25
Critical Tether Diameter [mm]		
0.35	0.525	0.7

**DAS 1.5.3** →  $P$ : impact probability of meteoroids and orbital debris with a tether effective X-sectional area

$$A_T = L_T (D_{TC} + d)$$

( $d$ : minimum fatal debris diameter)

FATAL IMPACT RATE [yr <sup>-1</sup> km <sup>-1</sup> ]			
800 km, 25°	8.38	4.31	2.55
800 km, 50°	8.80	4.56	2.69
800 km, 75°	10.41	5.50	3.26
1000 km, 25°	8.11	4.33	2.63
1000 km, 50°	8.52	4.56	2.78
1000 km, 75°	9.90	5.40	3.33
1400 km, 25°	5.30	2.37	1.35
1400 km, 50°	4.92	2.41	1.37
1400 km, 75°	5.30	2.64	1.53

**Fatal Impact Rate (FIR):**

$$FIR = -\frac{\ln(1-P)}{A_T \Delta t} A_T = -\frac{\ln(1-P)}{\Delta t}$$

# AI 19.1 Test #1

## Fatal Impact Rate Analysis

### Tether Diameter between 2.5 mm and 5 cm

Tether Diameter
2.5 mm
Minimum Fatal Debris Diameter
0.625 mm
Critical Tether Diameter
1.75 mm

**DAS 1.5.3** → for orbital debris in between 0.625 mm and 1 mm and meteoroids up to 5 mm

**CODRM-99R** → for orbital debris larger than 1 mm

Tether Diameter			
5.00 mm	1.00 cm	2.50 cm	5.00 cm
Minimum Fatal Debris Diameter			
1.25 mm	2.5 mm	6.25 mm	1.25 cm
Critical Tether Diameter			
3.5 mm	7 mm	1.75 cm	3.5 cm

**DAS 1.5.3** → for meteoroids up to 5 mm

**CODRM-99R** → for orbital debris

# AI 19.1 Test #1

## Fatal Impact Rate Analysis

### Tether Diameter between 2.5 mm and 5 cm

#### Minimum Fatal Debris Diameter

0.625 mm	1.25 mm	2.5 mm	6.25 mm	1.25 cm
----------	---------	--------	---------	---------

#### Tether Diameter

2.5 mm	5 mm	1 cm	2.5 cm	5 cm
--------	------	------	--------	------

#### Models to compute the differential debris flux

##### DAS 1.5.3

0.625 mm – 1 mm	-	-	-	-
-----------------	---	---	---	---

##### CODRM-99R

1 mm – 1 cm at intervals of 1 mm	1.25 mm – 2 mm 2 mm – 1 cm at intervals of 1 mm	2.5 mm – 3 mm 3 mm – 1 cm at intervals of 1 mm	6.25 mm – 7 mm 7 mm – 1 cm at intervals of 1 mm	
1 cm – 10 cm at interval of 1 cm	1 cm – 10 cm at interval of 1 cm	1 cm – 10 cm at interval of 1 cm	1 cm – 10 cm at interval of 1 cm	1.25 cm – 2 cm 2 cm – 10 cm at interval of 1 cm
10 cm – 1 m at intervals of 10 cm	10 cm – 1 m at intervals of 10 cm	10 cm – 1 m at intervals of 10 cm	10 cm – 1 m at intervals of 10 cm	10 cm – 1 m at intervals of 10 cm
Larger than 1m	Larger than 1m	Larger than 1m	Larger than 1m	Larger than 1m

# AI 19.1 Test #1

## Fatal Impact Rate Analysis

### Tether Diameter between 2.5 mm and 5 cm

---

#### Differential Fatal Impact Rate ( $d\_fir(i)$ )

$$d\_fir(i) = d\_flux(i) \times A_T(i)$$

$d\_flux(i)$ : **differential debris flux** (i.e. flux for each *ith* debris diameter interval)

$A_T(i)$ : tether effective cross-sectional area

For debris particles  $< 1$  m:

$$A_T(i) = L_T \times (D_{TC} + D(i))$$

$$D(i) = d_{min}(i) + \frac{d_{max}(i) - d_{min}(i)}{2}$$

$d_{min}(i)$  and  $d_{max}(i)$  minimum & maximum diameter of the debris interval *ith*

For space debris  $\geq 1$  m

$$d\_fir_{\geq 1m} = d\_flux_{\geq 1m} \times L_T \times 5$$

Assuming that 5 m is the average diameter of large space objects

The **total fatal impact rate (FIR)** is obtained by summing the differential impact rates on all the debris diameter intervals ( $N$ )

$$FIR = \sum_{i=1}^N d\_fir(i)$$

# AI 19.1 Test #1

## Fatal Impact Rate Analysis

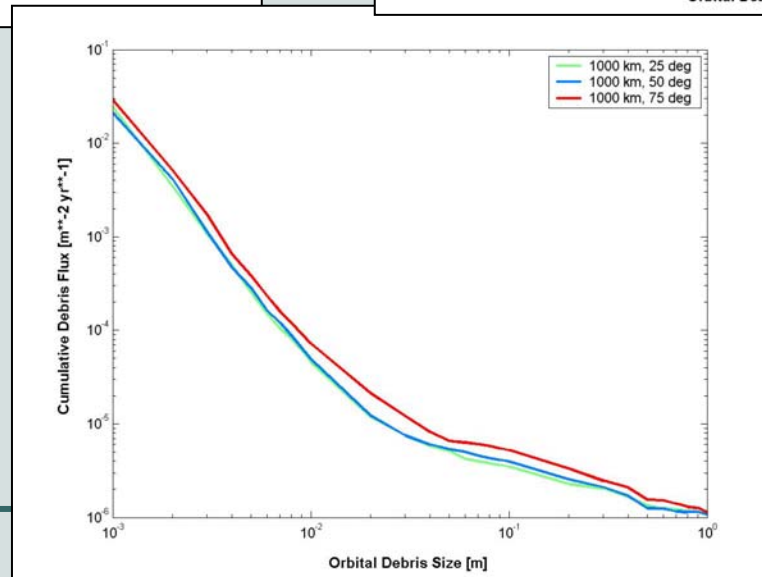
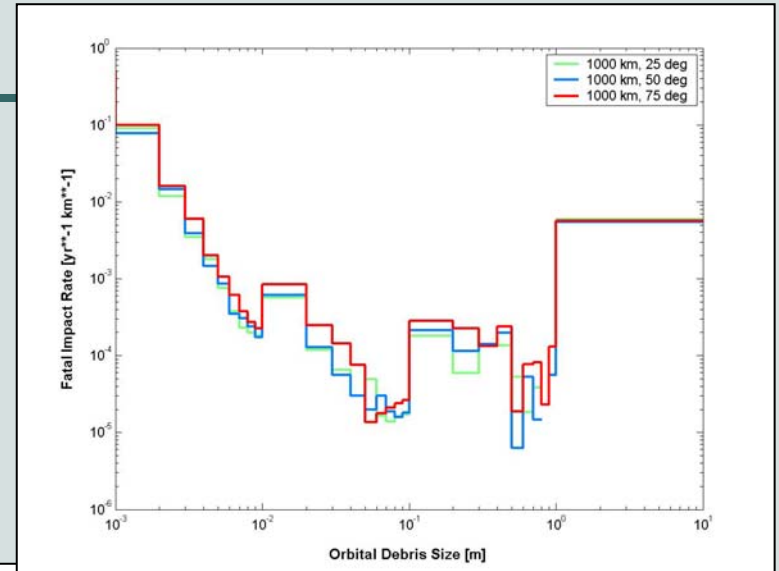
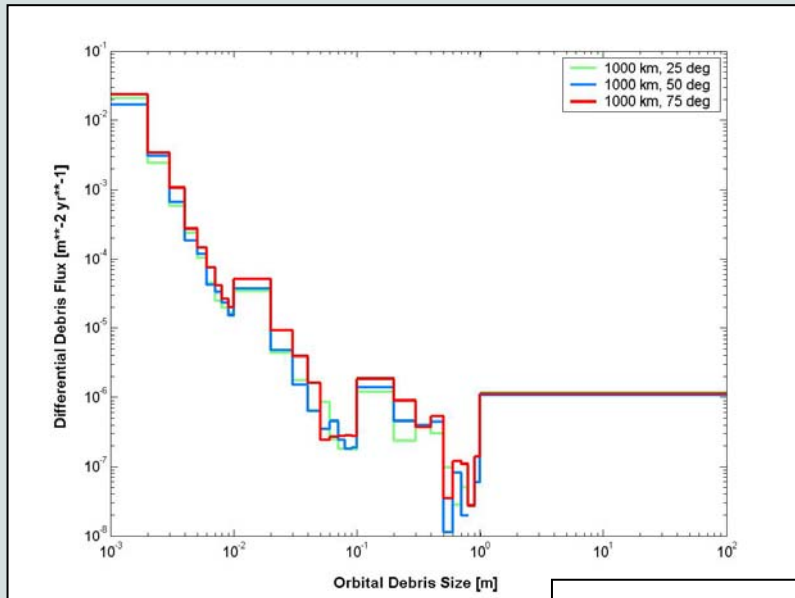
### Tether Diameter between 2.5 mm and 5 cm

---

Orbit Altitude & Inclination	Tether Diameter				
	2.50 mm	5.00 mm	1.00 cm	2.50 cm	5.00 cm
	<b>Fatal Impact Rate [yr<sup>-1</sup> km<sup>-1</sup>]</b>				
800 km, 25°	0.441	0.083	0.027	0.012	0.010
800 km, 50°	0.479	0.102	0.029	0.013	0.011
800 km, 75°	0.566	0.108	0.040	0.019	0.016
1000 km, 25°	0.491	0.104	0.027	0.010	0.008
1000 km, 50°	0.506	0.090	0.031	0.010	0.008
1000 km, 75°	0.625	0.102	0.040	0.013	0.009
1400 km, 25°	0.218	0.052	0.012	0.005	0.004
1400 km, 50°	0.237	0.061	0.015	0.006	0.005
1400 km, 75°	0.271	0.060	0.019	0.007	0.006

# AI 19.1 Test #1

## Fatal Impact Rate Analysis



# AI 19.1 Test #2

## Survivability Analysis

- Single-line tether diameter: 0.5 mm, 1 mm
- Double-line tether with diameter of 0.5 mm, 1 mm for each single wire and knots spaced at intervals of 100 m, 10 m, 5 m
- Tether Length: 7.5 km

Initial Altitude [km]	Orbit Inclination			
	0°	25°	50°	75°
	<b>DEORBIT TIME [days]</b>			
1400	170	220	325	TT not efficient at these altitudes and inclinations
1300	140	185	280	
1200	120	155	230	
1100	95	125	185	
1000	70	95	140	375
900	55	70	110	280
800	45	55	80	200
700	30	40	55	140
600	20	30	40	80
500	15	20	25	40
400	10	15	15	20

# AI 19.1 Test #2

## Survivability Analysis

### Mission De-orbiting Times

---

Altitude Interval [km]	Inclination			
	0°	25°	50°	75°
	DECAY TIME PER ALTITUDE SHELL [days]			
1400 – 1300	30	35	45	TT not efficient at these altitudes and inclinations
1300 – 1200	20	30	50	
1200 – 1100	25	30	45	
1100 – 1000	25	30	45	
1000 – 900	15	25	30	95
900 – 800	10	15	30	80
800 – 700	15	15	25	60
700 – 600	10	10	15	60
600 – 500	5	10	15	40
500 – 400	5	5	10	20
400 – 250	10	15	15	20

# AI 19.1 Test #2

## Survivability Analysis

### Environment Debris Model & Methods Adopted

---

The NASA DAS 1.5.3 Model was used to compute the impact probability of meteoroids and orbital debris on a single- or double-line tether

A numerical multi-step algorithm was developed at ISTI to estimate the survival/sever probability of a double-line tether during a de-orbiting mission.

FOR EACH ALTITUDE, in the middle of each altitude interval, it computes

1. the sever probability of a single line with length 100 m, 10 m or 5 m using DAS 1.5.3
2. the sever probability of two lines of the same tether segment
3. the survival probability of two lines of the same tether segment
4. the survival probability of the whole tether
5. The sever probability of the whole tether

Then, it estimates the total survival/sever probability for all the altitude intervals

# AI 19.1 Test #2

## Survivability Analysis

### Sever Probability of a Single-Line

---

**The sever probability**, or total probability of impact with space debris able to sever the tether, **of a single line** was computed with DAS 1.5.3 giving in input:

- Apogee/perigee altitude in the middle of each altitude shell (i.e. 1350/1350 km, 1250/1250 km, ... 350/350 km)
- Inclination of the initial orbit (i.e. 0°, 25°, 50°, 75°)
- Mission duration corresponding to the decay time in the selected altitude shell (i.e. 45 days, .., 15 days from 1400 km at 50°)
- Average tether effective cross-sectional area  $A_T = L_T (D_{TC} + d)$ , with  $d$  the minimum fatal debris diameter (i.e.  $d = 0.25 D_T$ ),  $L_T$  the tether length and  $D_{TC}$  the critical tether diameter (= 2 x 0.35 x  $D_T$ )
- Debris diameter  $d$



$$P_1(n,i)$$

$n$  is the  $n$ th tether segment

$i$  is the  $i$ th altitude interval

# AI 19.1 Test #2

## Survivability Analysis

### Sever Probability of the Tether at each Altitude

---

The sever probability of both lines of the same  $n$ th tether segment is

$$P_{SE}(n, i) = [P_1(n, i)]^2$$

And the corresponding survival probability is

$$P_{SU}(n, i) = 1 - P_{SE}(n, i)$$

The tether is severed if both lines of at least one segment of it are severed. But the tether survives if all segments survive. Then the survival probability of the whole tether at each altitude can be computed as

$$P_{SU\_T}(i) = \prod_{n=1}^N P_{SU}(n, i) = [P_{SU}(n, i)]^N$$

Where  $N$  is the number of tether segments, i.e.  $N = L_T/L$  and  $L$  is the length of a line:  
100 m, 10 m or 5 m

# AI 19.1 Test #2

## Survivability Analysis

### Sever Probability of the Tether During the De-orbiting Mission

---

The tether survives during the de-orbiting mission if it survives at each altitude. Then the **tether survival probability during the mission** is

$$P_{SU\_M} = \prod_{i=ih}^H P_{SU\_T}(i)$$

where  $H$  is the number of altitude intervals considered ( $H = 11$ ) and  $ih$  is the index to identify the starting altitude, i.e., if  $ih = 1$  corresponds to 1350 km,  $ih = 5$  to 950 km,..then:

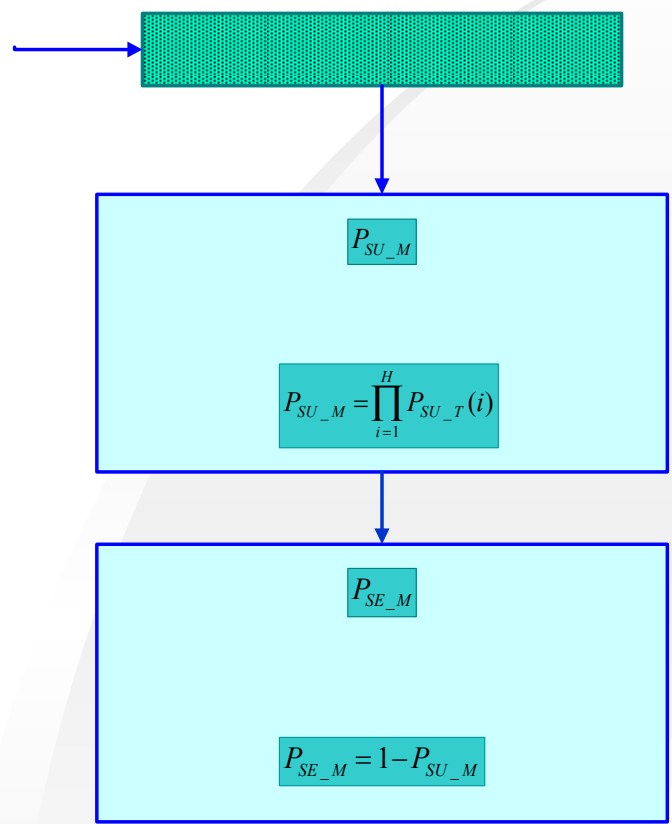
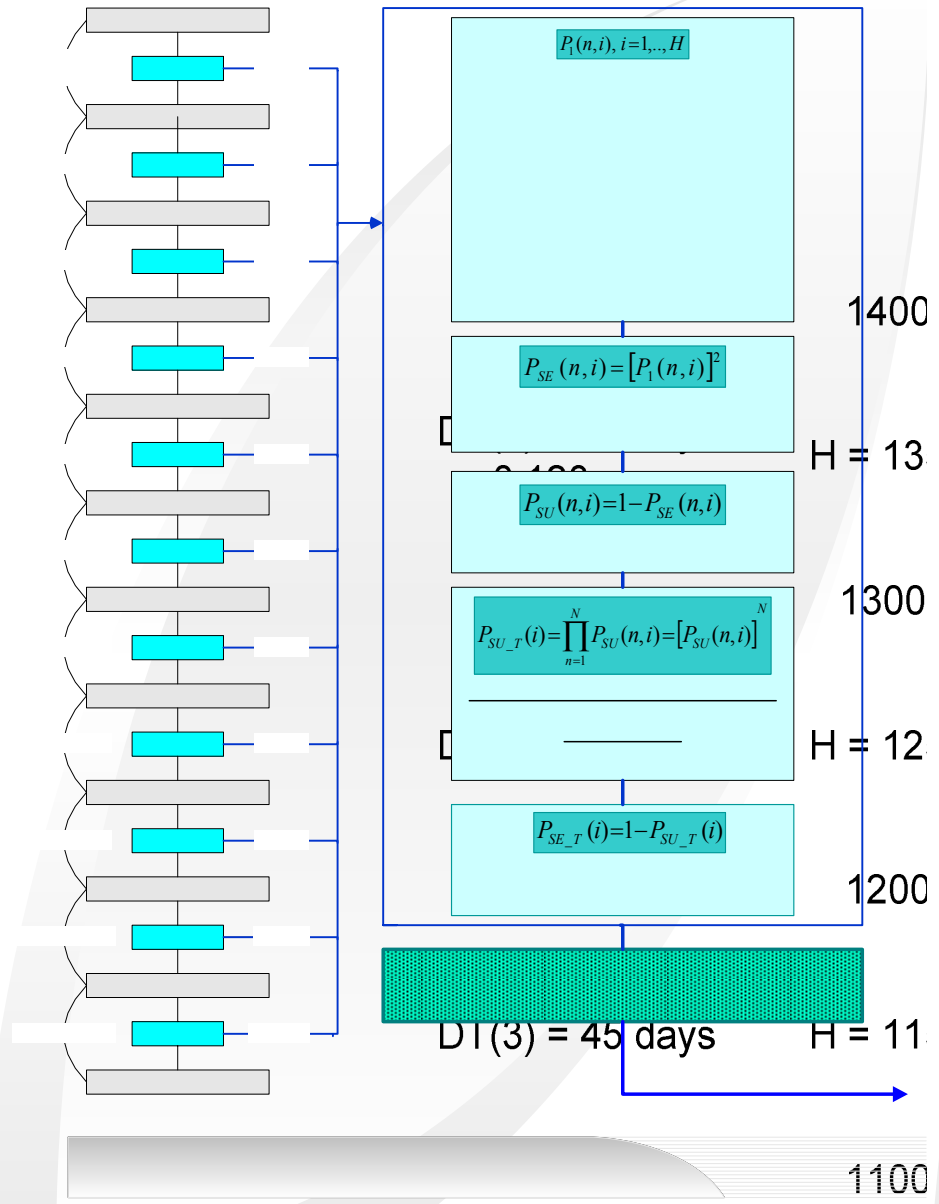
if de-orbiting is from 1400 km:  $ih = 1$

if de-orbiting is from 1000 km:  $ih = 5$

if de-orbiting is from 800 km:  $ih = 7$

The **tether sever probability during the mission** is

$$P_{SE\_M} = 1 - P_{SU\_M}$$



DT(4) = 45 days    H = 10

Apoge

A

# AI 19.1 Test #2

## Survivability Analysis

### Application: Impact Probability using DAS 1.5.3

Example of the **Impact Probability** of meteoroids and orbital debris with a tether segment line with diameter of 1 mm and length of 5 m. The tether X-sectional area is 0.00475 m<sup>2</sup>

Total Probability of Impact				
Altitude [km]	Inclination			
	0°	25°	50°	75°
1350	0.00051	0.00061	0.00079	-
1250	0.00034	0.00052	0.00088	-
1150	0.00046	0.00056	0.00086	-
1050	0.00057	0.00070	0.00109	-
950	0.00099	0.00168	0.00216	0.00825
850	0.00043	0.00066	0.00141	0.00455
750	0.00043	0.00044	0.00077	0.00223
650	0.00023	0.00024	0.00037	0.00174
550	0.00010	0.00021	0.00032	0.00097
450	0.00009	0.00009	0.00019	0.00041
350	0.00016	0.00024	0.00024	0.00034

# AI 19.1 Test #2

## Survivability Analysis

**Application: Survivability Probability at each Altitude**

**Tether of 7.5 km**  
**Knots spaced at intervals of 5 m**  
**Number of tether segments:  $7500/5 = 1500$**

<b>H</b>	<b>Inclination [deg]</b>			
	<b>0</b>	<b>25</b>	<b>50</b>	<b>75</b>
1350	0.99961	0.99944	0.99906	-----
1250	0.99983	0.99959	0.99884	-----
1150	0.99968	0.99953	0.99889	-----
1050	0.99951	0.99927	0.99822	-----
950	0.99853	0.99578	0.99303	0.90294
850	0.99972	0.99935	0.99702	0.96942
750	0.99972	0.99971	0.99911	0.99257
650	0.99992	0.99991	0.99979	0.99547
550	0.99999	0.99993	0.99985	0.99859
450	0.99999	0.99999	0.99995	0.99975
350	0.99996	0.99991	0.99991	0.99983

# AI 19.1 Test #2

## Survivability Analysis

### Application: Total Survivability Probability

---

Survival probability from 1400 km

0°	25°	50°	75°
0.99647	0.99243	0.98377	-

Survival probability from 1000 km

0°	25°	50°	75°
0.99783	0.99459	0.98869	0.86330

Survival probability from 800 km

0°	25°	50°	75°
0.99958	0.99946	0.99861	0.98626