Collision solutions in orbital elements space

G. B. Valsecchi[1], A. Rossi[2], A. Milani[3], S. R. Chesley[4]

[1]INAF-IASF, Roma (Italy)

[2]ISTI-CNR, Pisa (Italy)

[3]University of Pisa, Pisa (Italy)

[4]JPL, Pasadena (USA)

The problem

Impact monitoring programs find collision possibilities of NEAs with the Earth at specific dates, and characterize them with values of the stretching and of the impact probability. These depend on the observational data available for the NEA, and on assumptions on the statistical features of the data.

For short arcs, the confidence region can be large, and can contain collision solutions, that is virtual impactors.

Can we meaningfully speak of the size of a collision solution and, if yes, can we understand how does it vary for different NEA orbits?

This talk aims at convincing you that, under appropriate assumptions, the answer is yes to both questions.

Collision conditions

If we consider the Earth to be on a circular orbit of radius 1, then the conditions for a collision with the centre of the Earth are:

$$\begin{aligned} \frac{a(1-e^2)}{1 \pm \cos\omega} &= 1 \\ \Omega - \lambda_{\oplus,c} &= \frac{\pi}{2} \mp \frac{\pi}{2} \\ \omega + f_c &= \frac{\pi}{2} \mp \frac{\pi}{2}, \end{aligned}$$

where the upper sign is for collisions at the ascending node and $\lambda_{\oplus,c}$ is the longitude of the Earth at collision time.

Thus, collisions lie on a 3-dimensional manifold in the 6-dimensional space of orbital elements.

The Earth in the b-plane

In the b-plane of Öpik's theory, the collision condition is the point of coordinates $\xi = \zeta = 0$.

The impact radius of the Earth on that plane is:

$$b_\oplus = \sqrt{r_\oplus^2 + 2cr_\oplus};$$

where $c = m/U^2$, m is the ratio of the mass of the Earth to that of the Sun, $U = U(a, e, i)$ is the unperturbed geocentric speed of the NEA and $r_\oplus$ is the actual radius of the Earth.

We use $b_\oplus$ to compute the region in elements space corresponding to a physical collision.

Orbital elements & b-plane coordinates

If at $t = t_0$ the NEA is close to the Earth, with geocentric coordinates X_0, Y_0, Z_0, all very small, we have

$$\begin{aligned} \xi &= X_0 \cos\phi - Z_0 \sin\phi \\ \zeta &= (X_0 \sin\phi + Z_0 \cos\phi)\cos\theta - Y_0 \sin\theta, \end{aligned}$$

where $\theta = \theta(a, e, i)$ and $\phi = \phi(a, e, i)$.

We can express X_0, Y_0, Z_0 as functions of the orbital elements (to first order in X_0, Y_0, Z_0):

$$\begin{aligned} X_0 &= \frac{a(1-e^2)}{1 + e\cos f_0} - 1 \\ Y_0 &= \Omega - \lambda_\oplus + \arctan[\cos i \tan(\omega + f_0)] \\ Z_0 &= \sin i \sin(\omega + f_0). \end{aligned}$$

A generic collision

At epoch t^*, a NEA has orbital parameters a, e, i, ω, Ω, f^*; at $t = t_c$, when its true anomaly is f_c (and its mean anomaly is M_c), it has a collision with the Earth, located at $\lambda_{\oplus,c}$.

Note that $t_c = t^* + 2h\pi/n$, where h is the non-integer number of heliocentric revolutions made by the NEA between t^* and t_c, and $n = n(a)$ is mean motion of the NEA.

To understand what happens to ξ and ζ when we apply small changes to the orbital elements, we compute

$$d\xi = \sum_{i=1,6} \frac{\partial \xi}{\partial \mathcal{E}_i} d\mathcal{E}_i; \qquad d\zeta = \sum_{i=1,6} \frac{\partial \zeta}{\partial \mathcal{E}_i} d\mathcal{E}_i,$$

where $\mathcal{E}_1 = a, \mathcal{E}_2 = e, \mathcal{E}_3 = i, \mathcal{E}_4 = \omega, \mathcal{E}_5 = \Omega, \mathcal{E}_6 = M$.

Derivatives of ξ, ζ w.r.t. the elements

At collision, the derivatives $\partial\xi/\partial\mathcal{E}_i$, $\partial\zeta/\partial\mathcal{E}_i$ have the form:

$$
\begin{aligned}
\frac{\partial\xi}{\partial\mathcal{E}_i} &= \frac{\partial X}{\partial\mathcal{E}_i}\cos\phi - \frac{\partial Z}{\partial\mathcal{E}_i}\sin\phi \\
\frac{\partial\zeta}{\partial\mathcal{E}_i} &= \frac{\partial X}{\partial\mathcal{E}_i}\cos\theta\sin\phi + \frac{\partial Z}{\partial\mathcal{E}_i}\cos\theta\cos\phi - \frac{\partial Y}{\partial\mathcal{E}_i}\sin\theta,
\end{aligned}
$$

where terms with either X, or Y, or Z as factor have been dropped, since $X_c = Y_c = Z_c = 0$.

Among the derivatives, $\partial\xi/\partial a$ and $\partial\zeta/\partial a$ depend on h, the non-integer number of heliocentric revolutions made by the NEA between t^* and t_c.

Collision solution size

We can then write:

$$\begin{pmatrix} \delta\xi \\ \delta\zeta \end{pmatrix} = \begin{pmatrix} \frac{\partial\xi}{\partial a} & \frac{\partial\xi}{\partial e} & \frac{\partial\xi}{\partial\omega} & 0 & 0 \\ \frac{\partial\zeta}{\partial a} & \frac{\partial\zeta}{\partial e} & \frac{\partial\zeta}{\partial\omega} & \frac{\partial\zeta}{\partial\Omega} & \frac{\partial\zeta}{\partial M} \end{pmatrix} \begin{pmatrix} \delta a \\ \delta e \\ \delta\omega \\ \delta\Omega \\ \delta M \end{pmatrix},$$

since an explicit computation shows that
$\partial\xi/\partial i = \partial\xi/\partial\Omega = \partial\xi/\partial f = \partial\zeta/\partial i = 0$.

Fixing the values of three elements, in the plane of the two other elements we compute the ellipse corresponding to the b-plane circle centred in $(0, 0)$ of radius $b_\oplus$ (i.e., to the impact cross section of the Earth).

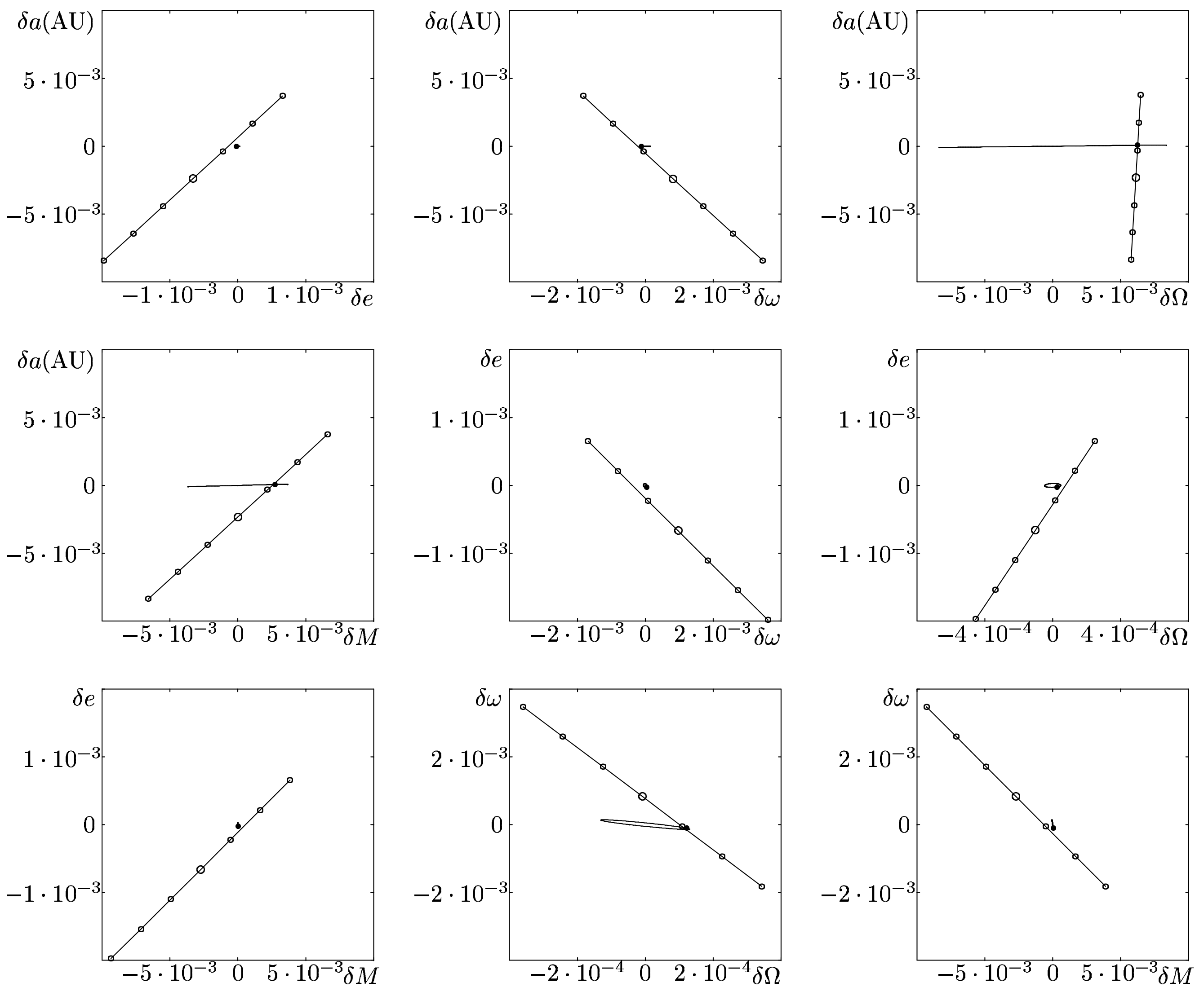
Application: 2002 NT_7 in 2019

In the summer of 2002 the observational record of 2002 NT_7 was compatible with a collision with the Earth taking place about $h = 7.3$ revolutions of the asteroid later, in early 2019.

Substituting the appropriate numeric values ($a = 1.74$, $e = 0.53$, $i = 42.3°$) in the expressions for $\delta\mathcal{E}_i$ we compute the collision region in various projections.

The next plot shows the collision in the δa-δe, δa-$\delta\omega$, δa-$\delta\Omega$, δa-δM, δe-$\delta\omega$, δe-$\delta\Omega$, δe-δM, $\delta\omega$-$\delta\Omega$, $\delta\omega$-δM planes. The collision region in each subplot is an ellipse, and the collision point found numerically is the full dot. The line represents the LoV, along which the $\sigma = -3, -2, -1, 1, 2, 3$ values are denoted with small open circles, and the nominal solution ($\sigma = 0$) is denoted by the large open circle.

Application: 2002 NT_7 in 2019



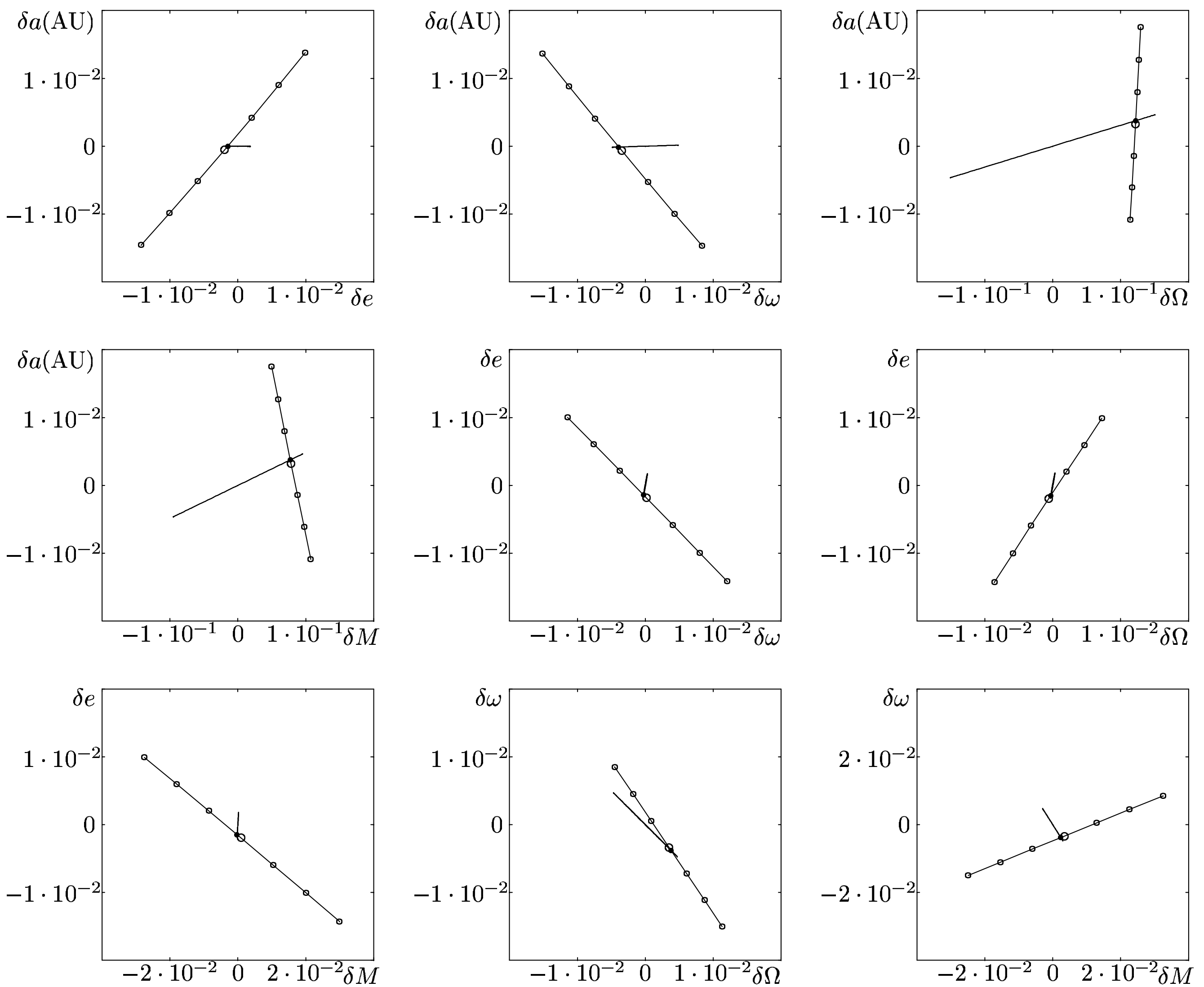
Application: 2003 EE_{16} in 2008

In March 2003 the the observational record of 2003 EE_{16} was compatible with a collision taking place about $h = 2.9$ revolutions of the asteroid later, in early 2008.

We can proceed as before, for $a = 1.42$, $e = 0.62$, $i = 0.6^\circ$, to get the 2008 collision region for 2003 EE_{16} in the various projections, as shown in the next plot.

As it is easily noticeable, the collision region for this low-inclination NEA is much larger than that of 2002 NT_7, as shown by the last plot in which the collision regions of both NEAs are plotted.

Application: 2003 EE_{16} in 2008



The two collision regions