

Modelling of Non-Stationary Autoregressive Alpha Stable Processes with Particle Filters

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Abstract—In this work, we propose a novel method to model time-varying autoregressive impulsive signals, which possess skewed or symmetric Alpha Stable distributions. The main contribution of this work is its ability to model both the unknown autoregressive coefficients and the distribution parameters, where all of them are time-varying. This method is a generalization of previous work by the same authors where the modelling of time-varying autoregressive coefficients, under constant distribution parameters was performed merely for symmetric distributions. The proposed method is composed of a particle filter, by which the time-varying autoregressive coefficients and the distribution parameters can be estimated successfully. The performance of the proposed method is tested for different parameter values where the time variation of the autoregressive coefficients and the distribution parameters are considered to be sinusoidal or piecewise constant in time. The location parameter is taken to be either zero or a ramp waveform. The successful performance of the proposed method serves as a promising contribution in the modelling of impulsive signals, which are frequently seen in many areas, such as teletraffic in computer communications, radar and sonar applications and mobile communications.

Index Terms— Skewed α -stable distributions, Bayesian estimation techniques, Particle filtering, Time varying autoregressive processes

This work is supported partially by TÜBİTAK-CNR project number: 102E027 and partially by Boğaziçi University Scientific Research Fund project number: 04A201. The first author was supported by NATO-TÜBİTAK A2 fellowship, throughout his research at ISTI-CNR, Italy.

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Abstract—In this work, we propose a novel method to model time-varying autoregressive impulsive signals, which possess skewed or symmetric Alpha Stable distributions. The main contribution of this work is its ability to model both the unknown autoregressive coefficients and the distribution parameters, where all of them are time-varying. This method is a generalization of previous work by the same authors where the modelling of time-varying autoregressive coefficients, under constant distribution parameters was performed merely for symmetric distributions. The proposed method is composed of a particle filter, by which the time-varying autoregressive coefficients and the distribution parameters can be estimated successfully. The performance of the proposed method is tested for different parameter values where the time variation of the autoregressive coefficients and the distribution parameters are considered to be sinusoidal or piecewise constant in time. The location parameter is taken to be either zero or a ramp waveform. The successful performance of the proposed method serves as a promising contribution in the modelling of impulsive signals, which are frequently seen in many areas, such as teletraffic in computer communications, radar and sonar applications and mobile communications.

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I. INTRODUCTION

Parametric signal modelling has been extensively utilized in many disciplines in order to understand the nature of the observed physical events. However, most of the effort on signal modelling was performed by forcing the assumption that signals are Gaussian distributed. This is widely preferred, since most of the parameter estimation techniques can be easily given in analytically tractable solutions under this assumption [1-3].

Although trying to fit normal distribution to the data seems reasonable in most of the cases that are encountered in the physical world from the Central Limit Theorem (CLT) [2] point of view, it should also be noted that such an assumption can lead to severe problems when impulsive signals are considered. For example, the tsunami waves generated after a major earthquake in the Pacific Ocean in December 2004; caused the death of thousands of people. Such a rise in sea level is a typical example for an outlier, which cannot be modelled by light tailed distributions, such as the Gaussian one.

Thus, in order to model such distributions with outliers, Alpha-Stable (α -stable) process modelling [4-8] has been developed in the literature which enabled the processing of heavy-tailed signals. In order to model these signals parametrically; Autoregressive (AR), Moving-Average (MA), Autoregressive Moving Average (ARMA) [6, 7, 9, 11] or nonlinear AR models [12] have been utilized. Among these, AR structure is widely used in teletraffic data modelling in computer communications [9]. This structure can also be used to parameterize a channel in telecommunications and can vary over time if the communications is performed in a wireless environment [10]. In this work, AR modelling of general α -stable processes is considered, where the α -stable distribution can be either skewed ($\beta \neq 0$), or symmetric ($\beta = 0$). In the physical world, skewed α -stable processes are widely encountered in many areas, such as the modelling of textures in image processing [13] and teletraffic data modelling in computer communications [5]. According to the parameters, two classes of estimation problem arise:

- a) AR parameters may be either time-invariant or time-varying,
- b) Distribution parameters of the alpha stable process may be either time-invariant or time-varying.

The case of time-invariant AR coefficients and distribution parameters are studied well in the literature, where Iteratively Reweighted Least Squares [12], Generalized Yule-Walker [6] and Markov Chain Monte Carlo (MCMC) based methods [14] are proposed to estimate the unknown parameters.

In case of time-varying AR (TVAR) coefficients with constant distribution parameters, Thavaneswaran and Peiris proposed a penalized minimum dispersion method in case of a presumably *known* shape parameter (α) of the α -stable process which is taken to be larger than one, i.e. $\alpha > 1$ [15]. Recently, Gençağa et. al. [16, 17] have proposed another technique, where particle filtering is used for the estimation of the TVAR coefficients, while a Hybrid Monte Carlo method is proposed to find the unknown, but constant distribution parameters, namely α and σ ,

of the Symmetric Alpha Stable (S α S) process.

In this work, the situation, where both the unknown AR coefficients and the distribution parameters are time-varying, is considered and a novel method is proposed to estimate all the unknown parameters. Here, the distribution parameters are also estimated by particle filtering, since these are also time-varying unknowns and thus cannot be found by batch-type of estimation techniques, such as MCMC or Hybrid Monte Carlo [16, 17].

Until now, methods regarding to α -stable processes are mentioned. Since α -stable distribution cannot be expressed by closed form probability density functions (pdf), except for some limited cases ($\alpha=2, \beta=0$ Gaussian; $\alpha=1, \beta=0$ Cauchy; $\alpha=0.5, \beta=-1$ Pearson), [16, 17] utilize numerical methods, when needed.

In addition to these, when the pdf of the studied signal is of a closed form, such as Mixture of Gaussians or Laplacian with **known** distribution parameters; the estimation of the unknown TVAR coefficients can be done successfully using Particle Filters [18, 19]. Apart from the signal modelling scheme, which is the case discussed so far, particle filters can also perform very well in signal enhancement, where a **Gaussian** signal is embedded in a S α S noise process [20]. In [20] the underlying **Gaussian** process has a TVAR structure to model a speech signal.

It should be noted that our method differs from [20] in the sense that a TVAR α -stable process is modelled here, i.e. both the unknown TVAR coefficients and the distribution parameters ($\alpha, \beta, \sigma_t, \delta_t$) of an α -stable process are estimated; whereas the main objective in [20] is to enhance a **Gaussian** TVAR signal, which is contaminated by a S α S noise.

Moreover, it should be noted that the proposed method differs from [16, 17] in the sense that the unknown distribution parameters are also time-varying beside the TVAR coefficients, modelling the most general case that can be witnessed.

In its most general form, particle filters enable us to obtain the optimal Bayesian solution of the systems that can be modelled by non-Gaussian and/or nonlinear state-space equations [21, 22]. For such systems, if the signals are non-stationary, particle filters can still provide us with the optimal Bayesian solution, since the estimation is performed sequentially. However, other Bayesian techniques, such as the MCMC [23], can only be used for stationary signals, since these methods have batch processing nature and discard the time information of the signals.

Motivated by these, a novel method is proposed here, which is capable of estimating both the TVAR coefficients and the time-varying distribution parameters of a general α -stable process. The results of the proposed method are

very promising to handle the most general modelling schemes, where the impulsive signals are involved. Such a modelling is of utmost importance, when the nature of the α -stable process passing through a time-varying, all-pass channel changes in time.

The rest of the paper is organized as follows: First, the problem is stated formally in Section II with background information on α -stable processes. Then, particle filters are introduced in Section III. In Section IV, the proposed method is presented which is followed by the experiments and discussions in Section V. Finally, conclusions are drawn in Section VI.

II. TVAR ALPHA STABLE PROCESSES

A. Alpha Stable Processes

It is well known by CLT that when a large number of random variables of different distributions are added, the summation variable tends to be more Gaussian distributed as the number of terms goes to infinity. Moreover, it is necessary that each added random variable is of finite variance. Otherwise, CLT becomes insufficient and Generalized CLT should be used [6]. In this case, the limiting distribution is an α -stable distribution. α -stable distributions are defined in terms of their characteristic functions, since their pdf cannot be obtained analytically, except for some limited cases ($\alpha=2$, $\beta=0$ Gaussian; $\alpha=1$, $\beta=0$ Cauchy; $\alpha=0.5$, $\beta=-1$ Pearson) [6, p. 18]. The characteristic function of α -stable distributions is given as follows:

$$\varphi(\tau) = \exp\left\{j\delta\tau - \sigma|\tau|^\alpha \left[1 + j\beta \text{sign}(\tau)\omega(\tau, \alpha)\right]\right\} \quad (1a)$$

Here, the parameters are defined within the following intervals: $-\infty < \delta < \infty$, $\sigma > 0$, $0 < \alpha \leq 2$, $-1 \leq \beta \leq 1$.

$$\omega(\tau, \alpha) = \begin{cases} \tan \frac{\alpha\pi}{2} & \text{if } \alpha \neq 1 \\ \frac{2}{\pi} \log|\tau| & \text{if } \alpha = 1 \end{cases} \quad \text{and} \quad \text{sign}(\tau) = \begin{cases} 1 & \text{if } \tau > 0 \\ 0 & \text{if } \tau = 0 \\ -1 & \text{if } \tau < 0 \end{cases} \quad (1b)$$

As shown above, an α -stable distribution is defined by four parameters and will be represented by $S(\alpha, \beta, \sigma, \delta)$, in the sequel. Among these, α and β are known as the shape and skewness parameters and they determine the thickness of

the tails and the symmetry of the distribution, respectively. As α gets smaller, the distributions become more impulsive. δ and σ are known as the measures of the location and the dispersion around it, respectively. For detailed information on stable distributions, the reader is referred to [7].

B. Problem Statement: Modelling of TVAR Alpha Stable Processes

As mentioned before, the main contribution of this work is the estimation of both the **time varying AR** coefficients and the distribution parameters of a general α -stable process, which is given in the following form:

$$y(t) = \sum_{k=1}^K x_k(t)y(t-k) + v(t) \quad (2)$$

where, $y(t)$, $x_k(t)$ are known as the observation process and autoregressive parameters, respectively. Here, $v(t)$ denotes the driving process, where its distribution is represented by $S(\alpha_t, \beta_t, \sigma_t, \delta_t)$. In this work, the objective is to estimate the TVAR coefficients, $x_k(t)$, and the distribution parameters of the α -stable process, i.e. α_t , β_t , σ_t and δ_t , where all of them depends on time index t .

III. PARTICLE FILTERS

Particle filters are known as the Sequential Monte Carlo methods, where the optimal Bayesian solution is obtained sequentially and the a posteriori distribution of the hidden variables in a non-Gaussian and/or nonlinear state-space modelling system can be provided. Such a system can be given by the following equations:

$$\begin{aligned} \boldsymbol{\theta}_t &= f_t(\boldsymbol{\theta}_{t-1}, \mathbf{v}_t) \\ \mathbf{y}_t &= h_t(\boldsymbol{\theta}_t, \mathbf{n}_t) \end{aligned} \quad (3)$$

where $\boldsymbol{\theta}_t$ and \mathbf{y}_t represent the hidden state and the observation vectors at current time t , respectively. Here, the process and observation noises are denoted by \mathbf{v}_t and \mathbf{n}_t , respectively. f_t and h_t are known as the process and observation functions and in their most general case, they are nonlinear. Also, the noise processes in (3) are modelled to be non-Gaussian. The aforementioned a posteriori distribution of the state variables is represented by

$p(\boldsymbol{\theta}_{0:t}|\mathbf{y}_{1:t})$, where $\mathbf{y}_{1:t}$ denotes the observation data. For (3), the optimal Bayesian solution for the a posteriori pdf is given as follows [21, 22]:

$$p(\boldsymbol{\theta}_{0:t}|\mathbf{y}_{1:t}) = \frac{p(\mathbf{y}_t|\boldsymbol{\theta}_t) p(\boldsymbol{\theta}_t|\boldsymbol{\theta}_{t-1})}{p(\mathbf{y}_t|\mathbf{y}_{t-1})} p(\boldsymbol{\theta}_{0:t-1}|\mathbf{y}_{1:t-1}) \quad (4)$$

In particle filtering, pdf's are expressed in terms of samples which are called as the particles. As a result of this, analytically inexpressible pdf's can also be handled by this numerical method. The expression for the a posteriori pdf can be given in terms of particles as follows:

$$p(\boldsymbol{\theta}_{0:t}|\mathbf{y}_{1:t}) \approx \sum_{i=1}^N w_t^i \delta(\boldsymbol{\theta}_{0:t} - \boldsymbol{\theta}_{0:t}^i) \quad (5)$$

where w_t^i , $\boldsymbol{\theta}_{0:t}^i$, $\delta(\cdot)$ denote the weight of the i^{th} particle, the i^{th} particle and the Kronecker delta operator, respectively. Once the pdf of a signal is obtained, quantities, such as the expectations, can be easily found by the appropriate integrations, as shown below:

$$I(f_t) = \int g(\boldsymbol{\theta}_{0:t}) p(\boldsymbol{\theta}_{0:t}|\mathbf{y}_{1:t}) d\boldsymbol{\theta}_{0:t} \quad (6)$$

where $g(\cdot)$ is a function depending on the estimate [21]. The integration given above can be calculated by drawing samples from the a posteriori pdf and then evaluating the value of the corresponding function, namely $g(\cdot)$, as shown below:

$$\hat{I}_N(f_t) = \sum_{i=1}^N g_t(\boldsymbol{\theta}_{0:t}^i) \tilde{w}_t^i \quad (7)$$

where \tilde{w}_t^i denote the normalized weights given as:

$$\tilde{w}_t^i = \frac{w_t^i}{\sum_{i=1}^N w_t^i}, \quad i=1, \dots, N \quad (8)$$

The particles that take place in equations (5) and (7) are drawn by a method known as the ‘‘Importance Sampling’’ [21, 22] and the corresponding ‘‘Importance Weight’’ for each of them is denoted by w_t^i as defined as follows:

$$w_t^i \propto \frac{p(\boldsymbol{\theta}_{0:t}^i | \mathbf{y}_{1:t})}{q(\boldsymbol{\theta}_{0:t}^i | \mathbf{y}_{1:t})} \quad (9)$$

where $q(\cdot)$ function is called as the ‘‘Importance Function’’ and drawing samples from this pdf is easier than that of the original distribution [21, 22]. However, importance sampling shown in (9), can be used in batch processing techniques and should be modified as follows for the sequential applications [21, 22]:

$$w_t^i \propto w_{t-1}^i \frac{p(\mathbf{y}_t | \boldsymbol{\theta}_t^i) p(\boldsymbol{\theta}_t^i | \boldsymbol{\theta}_{t-1}^i)}{q(\boldsymbol{\theta}_t^i | \boldsymbol{\theta}_{0:t-1}^i, \mathbf{y}_{1:t})} \quad (10)$$

Additionally, obtaining the optimal importance function $q(\cdot)$ is impossible most of the time and a simpler importance function, namely the a priori transition density function is used as a proposal density, which can be given as follows [21, 22]:

$$q(\boldsymbol{\theta}_t^i | \boldsymbol{\theta}_{0:t-1}^i, \mathbf{y}_{1:t}) \approx p(\boldsymbol{\theta}_t^i | \boldsymbol{\theta}_{t-1}^i) \quad (11)$$

Provided that this proposal distribution is used, the importance weight calculation (10), is reduced to the likelihood evaluation at the drawn sample value:

$$w_t^i \propto w_{t-1}^i p(y_t | \theta_t^i) \quad (12)$$

Then, normalized weights are calculated as in (8). This specific particle filtering scheme is known as the Bootstrap particle filter [21, 22].

Despite the implementation advantages of the Bootstrap particle filter, it usually performs unsatisfactorily because of neglecting the observation information in the process of sample transition [21, 22]. In literature, there are well known methods developed to alleviate this problem [24, 25].

If the a priori information about the states is not sufficient to construct a good approximation to the optimal importance function, one can also model the state transitions in such a way that the observation information is exploited not explicitly, but implicitly, by modelling the state transition noise with a Gaussian distribution, possessing a time-varying covariance matrix [18, 19].

IV. THE PROPOSED METHOD

An observed TVAR α -stable process can be expressed by the following equation:

$$y_t = \mathbf{y}_{t-1}^T \mathbf{x}_t + n_t \quad (13)$$

where $\mathbf{y}_{t-1} = [y_{t-1}, \dots, y_{t-K}]^T$ and $\mathbf{x}_t = [x_1(t), \dots, x_K(t)]^T$ vectors denote the past values of the AR process and the TVAR coefficient vector of order K , respectively. Here, $[\cdot]^T$ denotes vector transposition and the observation (driving) noise is modelled by an α -stable process, as shown below:

$$n_t \sim S(\alpha_t, \beta_t, \sigma_t, \delta_t) \quad (14)$$

In (13) and (14), index t denotes the time dependency of the AR coefficients and the distribution parameters.

The objective of this work, is to model an observed TVAR α -stable process y_t , i.e. to estimate the vector \mathbf{x}_t and the distribution parameters $(\alpha_t, \beta_t, \sigma_t, \delta_t)$, jointly. Here, the use of Bootstrap particle filter is proposed where the state vector is formed by augmenting the unknown TVAR coefficient vector with the unknown distribution parameters, which is denoted by $\boldsymbol{\theta}_t = [\mathbf{x}_t, \alpha_t, \beta_t, \varphi_t, \delta_t]^T$. In this modelling, $\varphi = \log \sigma$ is used since the dispersion parameter should always take positive values.

Since we have no information regarding the transition of the state variables, a random walk model is formed to model the time-evolution of the state variables, which is given below:

$$\boldsymbol{\theta}_t = \boldsymbol{\theta}_{t-1} + \mathbf{V}_t \quad (15)$$

where the process noise vector is represented by $\mathbf{V} = [\mathbf{v}_x^T, v_\alpha, v_\beta, v_\varphi, v_\delta]^T$ and modeled by a Gaussian distribution, i.e. $\mathbf{V} \sim N(\mathbf{0}, \mathbf{S}_{V_t})$. In order to obtain better estimates, the covariance of the first component of the process noise is modelled to be time-varying, which enables discounting of old measurements [18-19] during the learning process of the TVAR coefficients. On the other hand, the variances of the last four terms of the process noise vector are modelled by constants, since no closed form expressions can be obtained for these terms providing the discounting of old measurements as in [18]. Following this brief information, the covariance matrix of the process noise is proposed to be in the following form: $\mathbf{S}_{V_t} = \text{diag}(s_t^1, \dots, s_t^K, s_t^\alpha, s_t^\beta, s_t^\varphi, s_t^\delta)$, where $\text{diag}(\cdot)$ indicates a diagonal matrix. Here, the first K components of the main diagonal correspond to the variances of the elements of vector \mathbf{v}_x , while the last four components denote the variances of v_α , v_β , v_φ and v_δ , respectively. The elements of this covariance matrix are chosen as shown below, according to the aforementioned discussions:

$$\begin{aligned} s_t^k &= \left(\frac{1}{\xi} - 1 \right) \text{var}(x_k(t-1)), \quad k = 1, 2, \dots, K \\ s_t^\alpha &= \text{constant} \\ s_t^\beta &= \text{constant} \\ s_t^\varphi &= \text{constant} \\ s_t^\delta &= \text{constant} \end{aligned} \quad (16)$$

Here, ξ denotes the forgetting factor which is chosen between 0 and 1 and $\text{var}(x_k(t-1))$ represents the variance of the particles corresponding to the k^{th} AR coefficient at time $(t-1)$. The latter quantity can be calculated as follows from the particles:

$$\text{var}(x_k(t-1)) = \frac{1}{N} \sum_{i=1}^N \left[x_k^i(t-1) - \frac{1}{N} \sum_{n=1}^N x_k^n(t-1) \right]^2, \quad i, n = 1, 2, \dots, N \quad (17)$$

where N denotes the total number of particles used in the particle filter. As a result of the procedure that is given by (15) through (17), the state transition model is obtained; resulting in the following state-space formulation of the problem:

$$\begin{bmatrix} \mathbf{x}_t \\ \alpha_t \\ \beta_t \\ \varphi_t \\ \delta_t \end{bmatrix} = \begin{bmatrix} \mathbf{x}_{t-1} \\ \alpha_{t-1} \\ \beta_{t-1} \\ \varphi_{t-1} \\ \delta_{t-1} \end{bmatrix} + \begin{bmatrix} \mathbf{v}_t^x \\ v_t^\alpha \\ v_t^\beta \\ v_t^\varphi \\ v_t^\delta \end{bmatrix} \quad (18.a)$$

$$y_t = \mathbf{y}_{t-1}^T \mathbf{x}_t + n_t, \quad n_t \sim S(\alpha_t, \beta_t, \sigma_t, \delta_t) \quad (18.b)$$

After forming a state transition model as explained above, particles corresponding to each state variable can be drawn sequentially by using (18.a). The alleged particle drawing operation translates into the following choice of importance function, as shown below:

$$q(\boldsymbol{\theta}_t^i | \boldsymbol{\theta}_{0:t-1}^i, \mathbf{y}_{1:t}) \approx p(\boldsymbol{\theta}_t^i | \boldsymbol{\theta}_{t-1}^i) \quad (19.a)$$

$$\boldsymbol{\theta}_t^i \sim N(\boldsymbol{\theta}_{t-1}^i, \mathbf{S}_{V_t}), \quad i = 1, 2, \dots, N \quad (19.b)$$

where (19.b) implies sampling from a Gaussian distribution with mean $\boldsymbol{\theta}_{t-1}^i$ and covariance matrix \mathbf{S}_{V_t} . Given a state-transition model, which is proposed by the procedure described in (15) through (18.a) with the proposal density function of (19.a), the importance weight of each particle can be calculated by using the likelihood function, given by (12). It is well known that the pdf of an α -stable random variable, denoted by (14), can be estimated numerically by taking the inverse Fourier Transform of its characteristic function which is shown as follows:

$$p(n_t | \alpha_t, \beta_t, \sigma_t, \delta_t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \exp\{j\delta_t \tau - \sigma_t |\tau|^{\alpha_t} [1 + j\beta_t \text{sign}(\tau)\omega(\tau, \alpha_t)]\} \exp(jn\tau) d\tau \quad (20)$$

where $j = \sqrt{-1}$. In order to calculate the importance weight, pertaining to the i^{th} particle, (12) takes the following form as a result of the relationship of (13):

$$w_t^i \propto p(y_t | \boldsymbol{\theta}_t^i) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \exp \left\{ j \delta_t^i \tau - \sigma_t^i |\tau|^{\alpha_t^i} \left[1 + j \beta_t^i \text{sign}(\tau) \omega(\tau, \alpha_t^i) \right] \right\} \exp \left(j (y_t - \mathbf{y}_t^T \mathbf{x}_t^i) \tau \right) d\tau \quad (21)$$

After the calculation of the importance weight of each particle, these weights are normalized as in (8) and resampling is performed afterwards to avoid the degeneracy problem.

In order to summarize, the pseudo-code of the proposed method is given below.

PSEUDO-CODE

For $i = 1$ to N ,

1. INITIATION:

Draw samples from the initial distributions of the state variables:

$$\begin{aligned} \mathbf{x}_0^i &\sim N(\mathbf{m}_x, \mathbf{P}_x) \\ \alpha_0^i &\sim U(0, 2] \\ \beta_0^i &\sim U[-1, 1] \\ \varphi_0^i &\sim N(m_\varphi, P_\varphi) \rightarrow \sigma_0^i = \exp(\varphi_0^i) \\ \delta_0^i &\sim N(m_\delta, P_\delta) \end{aligned}$$

where $U(\cdot)$ denotes uniform distribution, while \mathbf{m} and \mathbf{P} denote the mean and covariance matrices of the Gaussian distributions. Note that, \mathbf{P}_x is a diagonal matrix whereas P_φ and P_δ are positive scalars.

For $t = 1$ to T , (T denotes the data length)

2. STATE TRANSITIONS:

Calculate the variance of each AR coefficient:

$$\text{var}(x_k(t-1)) = \frac{1}{N} \sum_{i=1}^N \left[x_k^i(t-1) - \frac{1}{N} \sum_{n=1}^N x_k^n(t-1) \right]^2, \quad k = 1, \dots, K; \quad i, n = 1, 2, \dots, N$$

Calculate the time-varying variances of the state-transition density and form \mathbf{S}_V matrix using (16) as follows:

$$s_t^k = \left(\frac{1}{\xi} - 1 \right) \text{var}(x_k(t-1)), \quad k = 1, 2, \dots, K, \quad s_t^\alpha = \text{constant}, s_t^\beta = \text{constant}, s_t^\varphi = \text{constant}, s_t^\delta = \text{constant}$$

Draw new particles for each state variable by using the proposed state-transition equation:

$$\begin{bmatrix} \mathbf{x}_t \\ \alpha_t \\ \beta_t \\ \varphi_t \\ \delta_t \end{bmatrix} = \begin{bmatrix} \mathbf{x}_{t-1} \\ \alpha_{t-1} \\ \beta_{t-1} \\ \varphi_{t-1} \\ \delta_{t-1} \end{bmatrix} + \begin{bmatrix} \mathbf{v}_t^x \\ v_t^\alpha \\ v_t^\beta \\ v_t^\varphi \\ v_t^\delta \end{bmatrix}$$

$$\boldsymbol{\theta}_t^i \sim N(\boldsymbol{\theta}_{t-1}^i, \mathbf{S}_{v_t^i}), \quad i = 1, 2, \dots, N$$

where the following condition must be satisfied during the transition of states, namely α_t and β_t :

$$p(\alpha_t | \alpha_{t-1}) = N(\alpha_{t-1}, s_t^\alpha) I_{(0,2]}(\alpha_t), \quad p(\beta_t | \beta_{t-1}) = N(\beta_{t-1}, s_t^\beta) I_{[-1,1]}(\beta_t)$$

where I denotes the indicator function: $I_{[a,b]}(x) = \begin{cases} 1, & x \in [a, b] \\ 0, & x \notin [a, b] \end{cases}$

3. CALCULATE THE IMPORTANCE WEIGHT OF EACH PARTICLE:

$$w_t^i \propto p(y_t | \boldsymbol{\theta}_t^i) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \exp \left\{ j\delta_t^i \tau - \sigma_t^i |\tau| \alpha_t^i \left[1 + j\beta_t^i \text{sign}(\tau) \omega(\tau, \alpha_t^i) \right] \right\} \exp \left(j(y_t - \mathbf{y}_t^T \mathbf{x}_t^i) \tau \right) d\tau$$

4. NORMALIZE THE WEIGHTS:

$$\tilde{w}_t^i = \frac{w_t^i}{\sum_{i=1}^N w_t^i}$$

5. RESAMPLE

6. GO TO STEP 2.

V. EXPERIMENTS AND DISCUSSIONS

In this section, performance of the proposed method is shown by computer simulations, where different experiments with respect to the time-variation of the AR coefficients and the distribution parameters are performed. In both experiments, 20 realizations of first order TVAR α -stable processes are synthetically generated by (13) and (14) and ensemble averaged results are illustrated in the following figures for the estimations of the AR coefficients

and the distribution parameters, for each scenario. For the sake of completeness, the scenario is expressed as follows:

$$y_t = y_{t-1}^T x_t + n_t \quad (22)$$

where the objective is to estimate the time waveforms of the following state variables:

$\boldsymbol{\theta}_t = [\theta_1(t), \theta_2(t), \theta_3(t), \theta_4(t), \theta_5(t)]^T = [x_t, \alpha_t, \beta_t, \sigma_t, \delta_t]^T$. To measure the estimation performance numerically, the

Normalized Mean Square Error (NMSE) of each state variable is also estimated by the following equation:

$$NMSE(t) = \frac{\sum_{i=1}^{20} (\hat{\theta}_{p,r}(t) - \theta_{p,r}(t))^2}{\sum_{r=1}^{20} \sum_{t=1}^M \theta_{p,r}^2(t)}, \quad p = 1, 2, 3, 4, 5 \quad (23)$$

where $\hat{\theta}$ and r denote the estimate and the member index of the waveform in the ensemble, respectively. In all experiments, M represents the length of the observed data, which is taken to be 3000. The details of the experiments are illustrated in Table 1.

A. Sequential Modelling of TVAR SaS Processes

In this section, the proposed method is used to model TVAR SaS processes. Thus, parameter β is taken to be zero during the experiments performed here.

A1. Experiment 1:

In this experiment, the TVAR coefficient, the shape and dispersion parameters of the α -stable process are taken to be piecewise constants in time, where all of them change their values at $t = M/2$. This is illustrated below:

$$x_t = \begin{cases} 0.9 & t < M/2 \\ 0.5 & t \geq M/2 \end{cases}, \alpha_t = \begin{cases} 1.5 & t < M/2 \\ 1.1 & t \geq M/2 \end{cases}, \sigma_t = \begin{cases} 2 & t < M/2 \\ 5 & t \geq M/2 \end{cases}, \beta = 0, \delta = 0 \quad (24)$$

By using the first expression in (16), the variance of the first component of matrix \mathbf{S}_{V_t} is calculated as follows:

$$s_t^1 = \text{var}(v_t^x) = \left(\frac{1}{\xi} - 1 \right) \text{var}(x(t-1)) \quad (25)$$

where the following values are used:

$$\xi = 0.9, \quad s_t^\alpha = 5 \times 10^{-4}, \quad s_t^\varphi = 5 \times 10^{-4} \quad (26)$$

On the first column of Fig. 1., estimates of x_t , α_t and σ_t are shown, respectively. Then, their corresponding NMSE curves are illustrated on the second column.

A2. Experiment 2:

In this experiment, AR coefficient and the shape parameter are chosen to be piecewise constant as in Experiment A1, while the variation of the dispersion parameter is considered to be sinusoidally changing in time, as shown below:

$$\sigma_t = 1.5 + \sin\left(\frac{2\pi t}{M}\right) \quad (27)$$

The first column of Fig. 2. illustrates the estimates of x_t , α_t and σ_t , while their corresponding NMSE curves are illustrated on the second column of Fig. 2.

B. Sequential Modelling of TVAR Skewed α -stable Processes

In this section, the proposed method is used to model TVAR Skewed α -stable processes. Thus, parameter β is not zero during the experiments and changes in time.

B1. Experiment 1:

In this experiment, the TVAR coefficient, the shape and dispersion parameters of the α -stable process are as in Experiment A2. Additionally, the skewness parameter is also taken to be time-varying as shown below:

$$\beta_t = \begin{cases} 0.5 & t < M/2 \\ -0.5 & t \geq M/2 \end{cases} \quad (28)$$

The first column of Fig. 3. illustrates the estimates of x_t , α_t , β_t and σ_t , while their corresponding NMSE curves are illustrated on the second column of Fig. 3.

B2. Experiment 2:

In this experiment, the TVAR coefficient, the shape, dispersion and skewness parameters of the α -stable process are taken as in the previous experiment. Moreover, the location parameter is also modelled here, where it is taken to be a ramp during the generation of the synthetic data. This is illustrated below:

$$\delta = \frac{1}{M}t \quad (29)$$

The first column of Fig. 4. illustrates the estimates of x_b , α_b , β_b , σ_t and δ_t , while their corresponding NMSE curves are illustrated on the second column of Fig. 4.

B3. Experiment 3:

In this experiment, the TVAR coefficient, the shape, skewness and location parameters of the α -stable process are taken as in the previous case. However, the variation of the dispersion parameter is taken to be piecewise continuous as illustrated below:

$$\sigma_t = \begin{cases} 2 & t < M/2 \\ 1 & t \geq M/2 \end{cases} \quad (30)$$

The first column of Fig. 5. illustrates the estimates of x_b , α_b , β_b , σ_t and δ_t , while their corresponding NMSE curves are illustrated on the second column of Fig. 5.

C. Sequential Modelling of TVAR S α S Processes (Sinusoidal AR variation)

In this section, the proposed method is used to model TVAR S α S processes. Thus, parameter β is taken to be zero during the experiments performed here.

C1. Experiment 1:

In this experiment, AR coefficient is taken to be sinusoidally varying in time, as shown below:

$$x_t = \sin\left(\frac{2\pi t}{M}\right) \quad (31)$$

while the variation of the shape and dispersion parameters are selected to be as in Experiment B3.

The first column of Fig. 6. illustrates the estimates of x_b , α_t and σ_t , while their corresponding NMSE curves are illustrated on the second column of Fig. 6.

As a result of the simulations, it is observed that successful estimates of the TVAR and distribution parameters can be obtained by the proposed method. This method can serve as a promising solution to track different impulsive signals passing through time-varying communication channels. From the simulations, it can be concluded that, the tracking capability of the proposed particle filtering scheme is good enough to model the instantaneously changing distribution parameters. It should be noted that if the location parameter is also estimated in addition to the other parameters, the quality of this parameter degrades when compared with the others, which is also a common observation in the other works done for time-invariant α -stable distributions in literature [see 7 and the references therein].

VI. CONCLUSIONS

In this work, a novel method is proposed to estimate the TVAR coefficients and the time-varying distribution parameters of a general α -stable process, jointly. This is a generalization of the method that is developed for S α S processes with TVAR coefficients but constant distribution parameters [16, 17]. Skewed α -stable processes with time-varying AR and distribution parameters can be estimated by the proposed method.

The proposed algorithm is composed of a particle filter where the time-varying AR coefficients and the distribution parameters are modelled by a state vector and a state-transition model is presented in order to approximate the optimal importance function.

In order to model TVAR S α S processes with constant distribution parameters, it is a reasonable decision to prefer methods which utilize MCMC techniques [16, 17], since they provide estimates with lower variances when compared to the particle filtering techniques. On the other hand, if the distribution parameters are also time-varying beside the AR coefficients, then one cannot refer to methods that are variants of MCMC, since the time information is neglected in those kinds of methods with batch data processing nature. However, these situations can be handled by the proposed method, successfully.

In conclusion, the successful performance of the developed method serves as a promising contribution in the modelling of time-varying impulsive signals, which are frequently seen in many areas, such as teletraffic in computer communications, radar and sonar applications and mobile communications, especially when it is needed to track the changing nature of the signals.

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TABLE 1: Experimental Scenarios

x_t : AR coefficient, s_t^1 : Variance of the process noise term corresponding to the transition of state variable x_t

α : Shape parameter, s_t^α : Variance of the process noise term corresponding to the transition of state variable α

β : Skewness parameter, s_t^β : Variance of the process noise term corresponding to the transition of state variable β

σ : Dispersion parameter, s_t^φ : Variance of the process noise term corresponding to the transition of state variable $\varphi = \log \sigma$

δ : Location parameter: s_t^δ : Variance of the process noise term corresponding to the transition of state variable δ

The variance terms given above are defined in (16).

Exp	AR	α	β	σ	δ
A1	$x_t = \begin{cases} 0.9 & t < M/2 \\ 0.5 & t \geq M/2 \end{cases}$ $s_t^1 = \left(\frac{1}{\xi} - 1\right) \text{var}(x(t-1))$	$\alpha_t = \begin{cases} 1.5 & t < M/2 \\ 1.1 & t \geq M/2 \end{cases}$ $s_t^\alpha = 5 \times 10^{-4}$	0	$\sigma_t = \begin{cases} 2 & t < M/2 \\ 5 & t \geq M/2 \end{cases}$ $s_t^\varphi = 5 \times 10^{-4}$	0
A2	$x_t = \begin{cases} 0.9 & t < M/2 \\ 0.5 & t \geq M/2 \end{cases}$ $s_t^1 = \left(\frac{1}{\xi} - 1\right) \text{var}(x(t-1))$	$\alpha_t = \begin{cases} 1.5 & t < M/2 \\ 1.1 & t \geq M/2 \end{cases}$ $s_t^\alpha = 5 \times 10^{-4}$	0	$\sigma_t = 1.5 + \sin\left(\frac{2\pi t}{M}\right)$ $s_t^\varphi = 5 \times 10^{-3}$	0
B1	$x_t = \begin{cases} 0.9 & t < M/2 \\ 0.5 & t \geq M/2 \end{cases}$ $s_t^1 = \left(\frac{1}{\xi} - 1\right) \text{var}(x(t-1))$	$\alpha_t = \begin{cases} 1.5 & t < M/2 \\ 1.1 & t \geq M/2 \end{cases}$ $s_t^\alpha = 5 \times 10^{-4}$	$\beta_t = \begin{cases} 0.5 & t < M/2 \\ -0.5 & t \geq M/2 \end{cases}$ $s_t^\beta = 5 \times 10^{-3}$	$\sigma_t = 1.5 + \sin\left(\frac{2\pi t}{M}\right)$ $s_t^\varphi = 5 \times 10^{-3}$	0
B2	$x_t = \begin{cases} 0.9 & t < M/2 \\ 0.5 & t \geq M/2 \end{cases}$ $s_t^1 = \left(\frac{1}{\xi} - 1\right) \text{var}(x(t-1))$	$\alpha_t = \begin{cases} 1.5 & t < M/2 \\ 1.1 & t \geq M/2 \end{cases}$ $s_t^\alpha = 5 \times 10^{-4}$	$\beta_t = \begin{cases} 0.5 & t < M/2 \\ -0.5 & t \geq M/2 \end{cases}$ $s_t^\beta = 5 \times 10^{-3}$	$\sigma_t = 1.5 + \sin\left(\frac{2\pi t}{M}\right)$ $s_t^\varphi = 5 \times 10^{-3}$	$\delta = \frac{1}{M} t$ $s_t^\delta = 5 \times 10^{-3}$
B3	$x_t = \begin{cases} 0.9 & t < M/2 \\ 0.5 & t \geq M/2 \end{cases}$ $s_t^1 = \left(\frac{1}{\xi} - 1\right) \text{var}(x(t-1))$	$\alpha_t = \begin{cases} 1.5 & t < M/2 \\ 1.1 & t \geq M/2 \end{cases}$ $s_t^\alpha = 5 \times 10^{-4}$	$\beta_t = \begin{cases} 0.5 & t < M/2 \\ -0.5 & t \geq M/2 \end{cases}$ $s_t^\beta = 5 \times 10^{-3}$	$\sigma_t = \begin{cases} 2 & t < M/2 \\ 1 & t \geq M/2 \end{cases}$ $s_t^\varphi = 5 \times 10^{-4}$	$\delta = \frac{1}{M} t$ $s_t^\delta = 5 \times 10^{-3}$
C1	$x_t = \sin\left(\frac{2\pi t}{M}\right)$ $s_t^1 = \left(\frac{1}{\xi} - 1\right) \text{var}(x(t-1))$	$\alpha_t = \begin{cases} 1.5 & t < M/2 \\ 1.1 & t \geq M/2 \end{cases}$ $s_t^\alpha = 5 \times 10^{-4}$	0	$\sigma_t = \begin{cases} 2 & t < M/2 \\ 1 & t \geq M/2 \end{cases}$ $s_t^\varphi = 5 \times 10^{-4}$	0

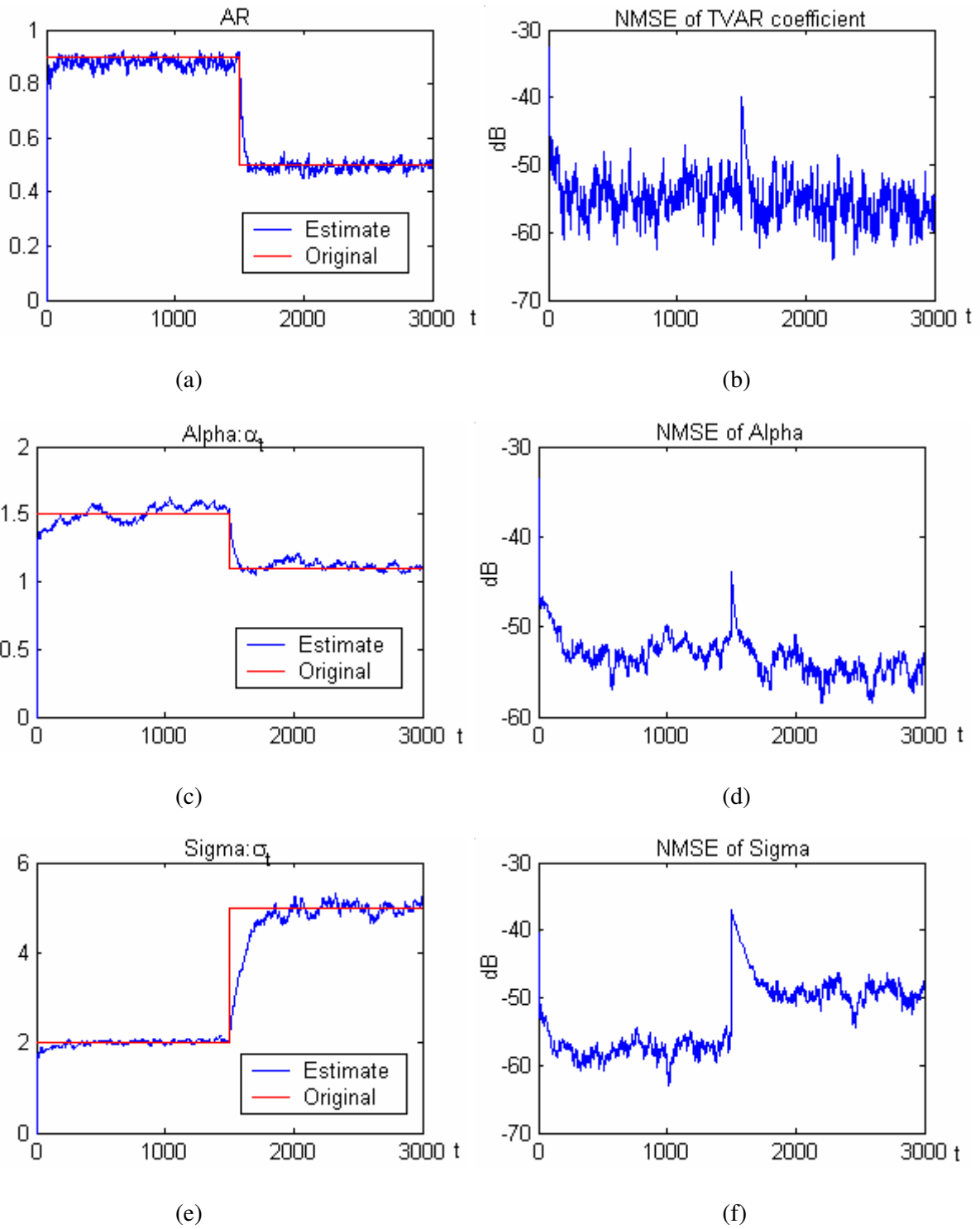


Fig. 1. Experiment A1: Estimation of time-varying AR coefficient and distribution parameters of *SaS* process

a) Estimation of TVAR coefficient, b) NMSE curve of TVAR estimate, c) Estimation of the shape parameter, d) NMSE curve of the shape parameter estimation, e) Estimation of the dispersion parameter, f) NMSE curve of the dispersion parameter estimation

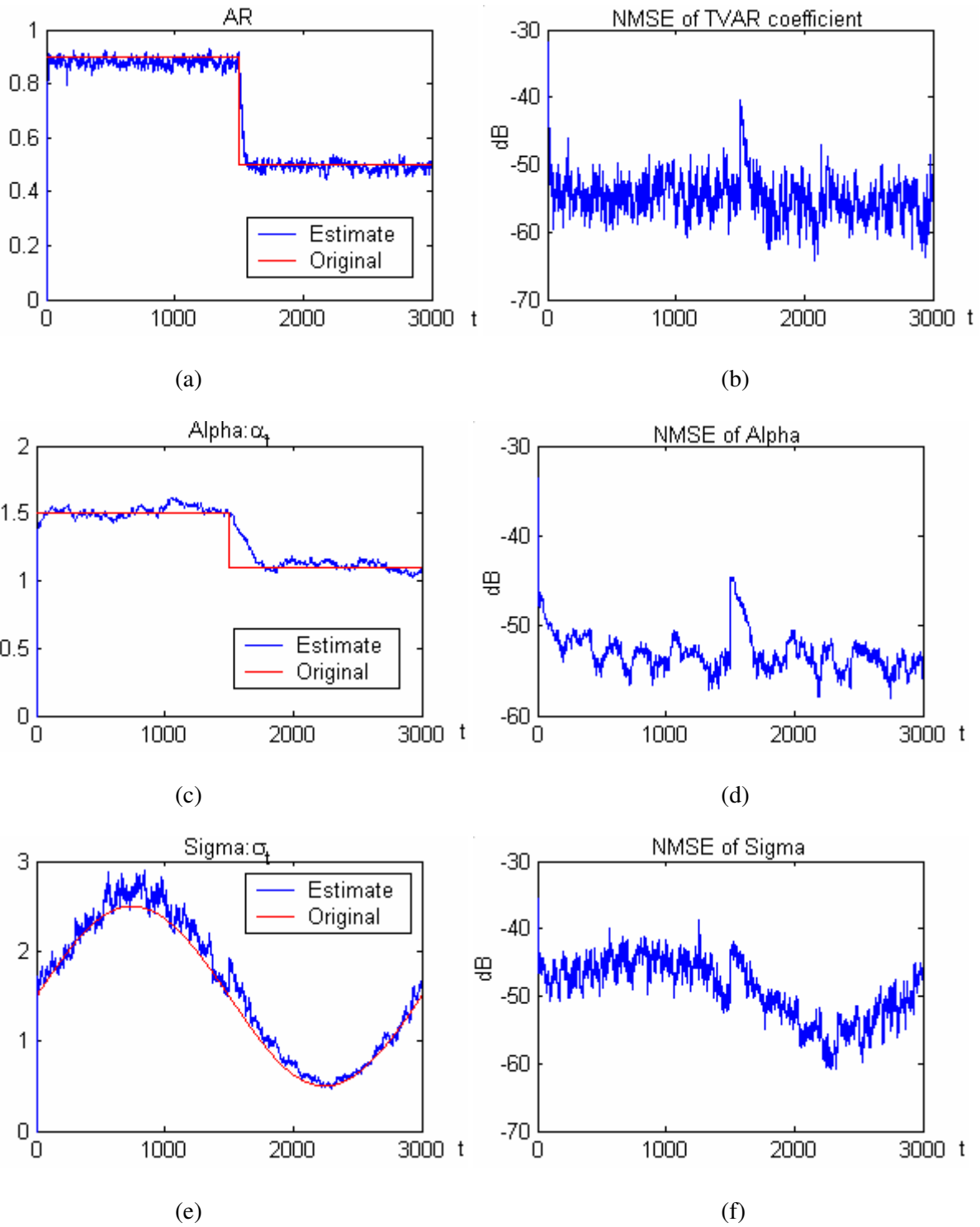


Fig. 2. Experiment A2: Estimation of time-varying AR coefficient and distribution parameters of *SaS* process

a) Estimation of TVAR coefficient, b) NMSE curve of TVAR estimate, c) Estimation of the shape parameter, d) NMSE curve of the shape parameter estimation, e) Estimation of the dispersion parameter, f) NMSE curve of the dispersion parameter estimation

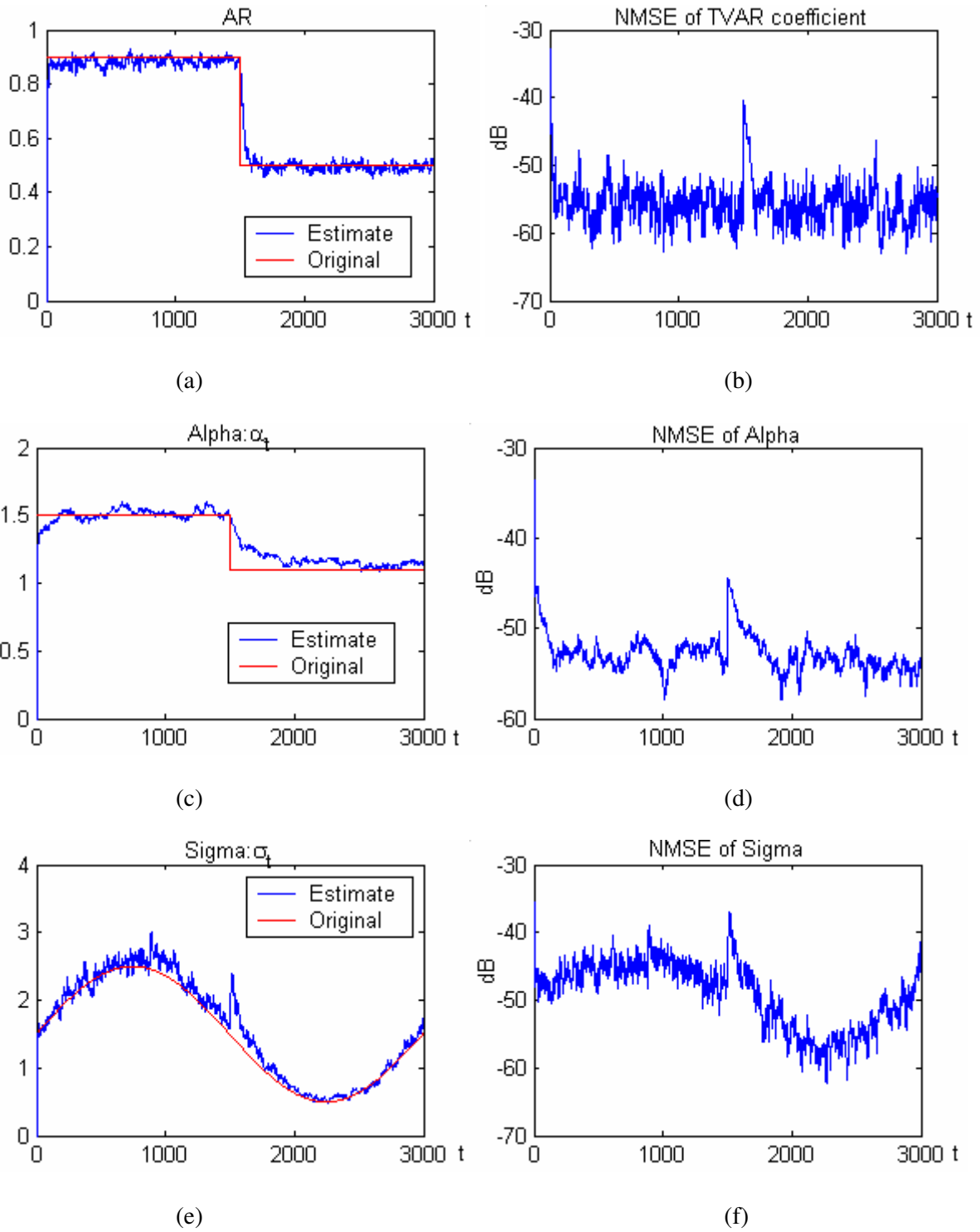


Fig. 3. Experiment B1: Estimation of time-varying AR coefficient and distribution parameters of *Skewed α -stable* process

- a) Estimation of TVAR coefficient, b) NMSE curve of TVAR estimate, c) Estimation of the shape parameter, d) NMSE curve of the shape parameter estimation, e) Estimation of the dispersion parameter, f) NMSE curve of the dispersion parameter estimation

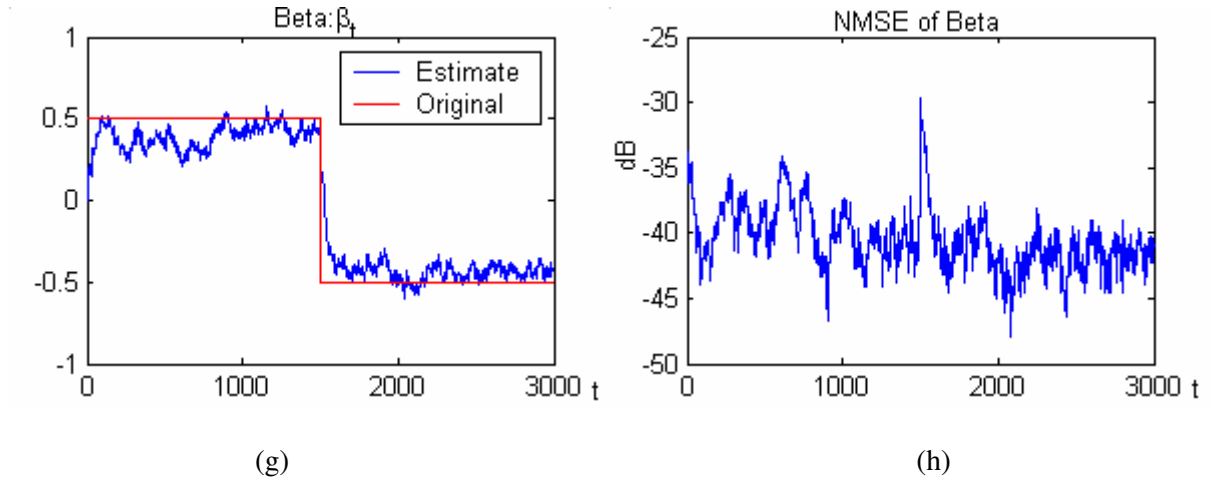


Fig. 3. (continued) g) Estimation of the skewness parameter, h) NMSE curve of the skewness parameter estimation

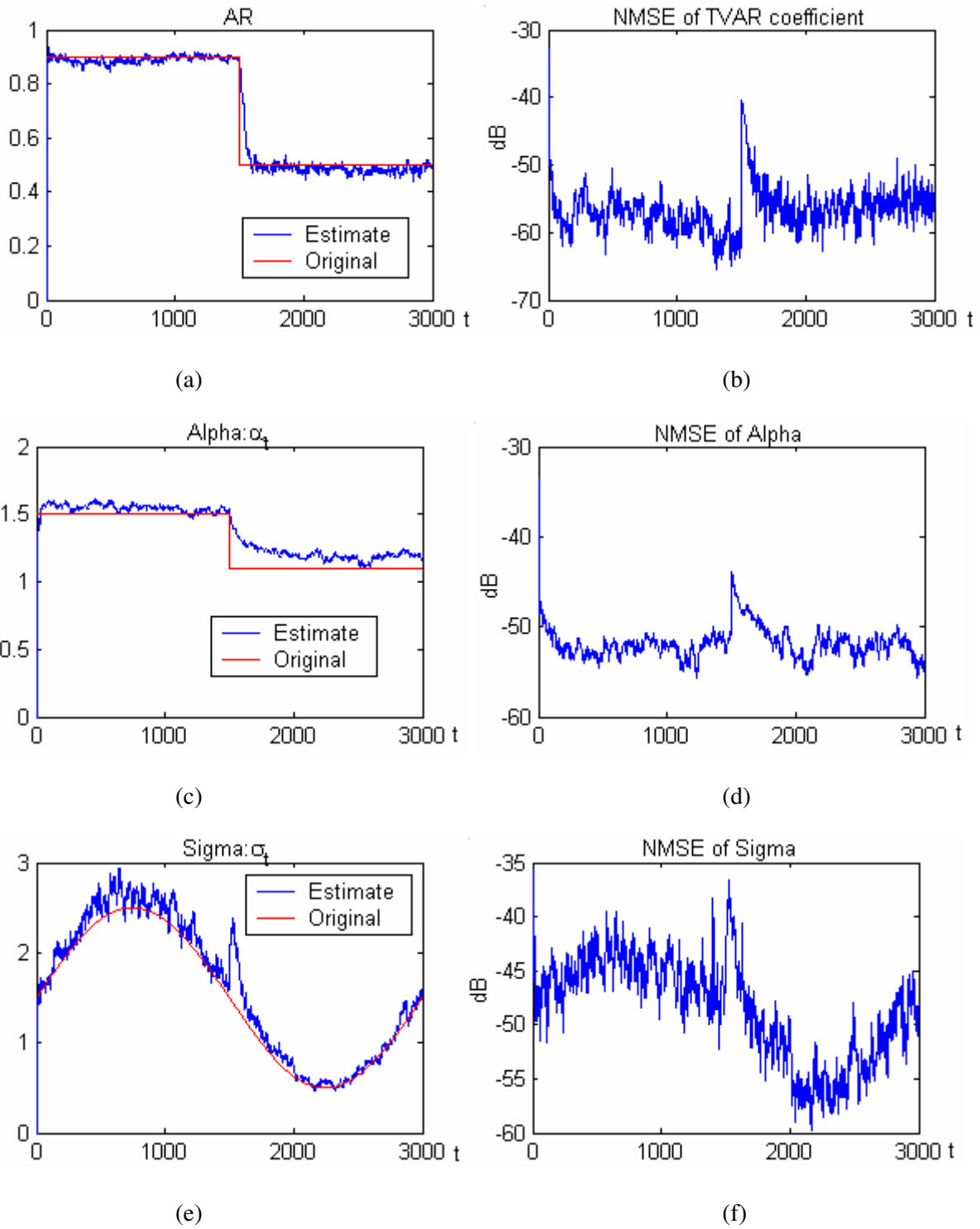
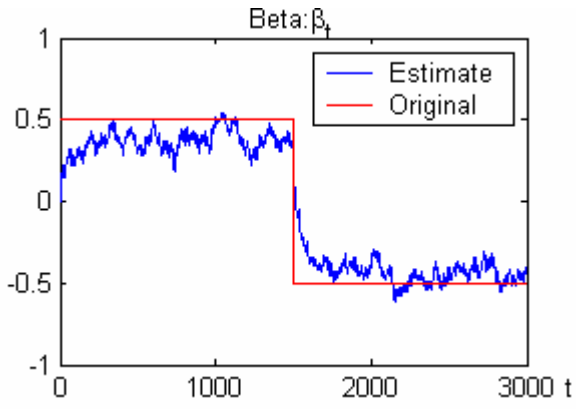
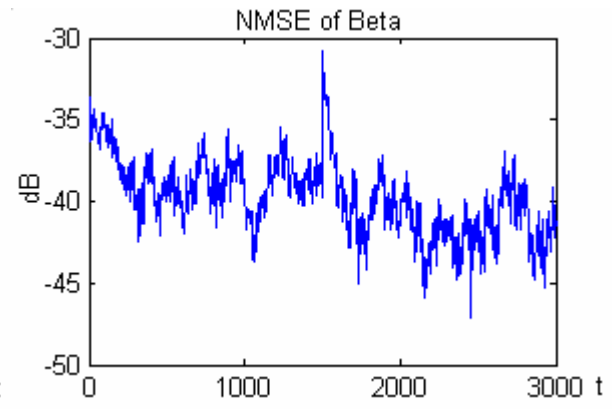


Fig. 4. Experiment B2: Estimation of time-varying AR coefficient and distribution parameters of *Skewed α -stable* process

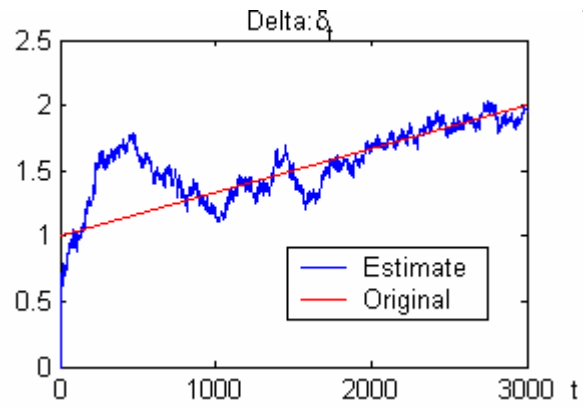
a) Estimation of TVAR coefficient, b) NMSE curve of TVAR estimate, c) Estimation of the shape parameter, d) NMSE curve of the shape parameter estimation, e) Estimation of the dispersion parameter, f) NMSE curve of the dispersion parameter estimation,



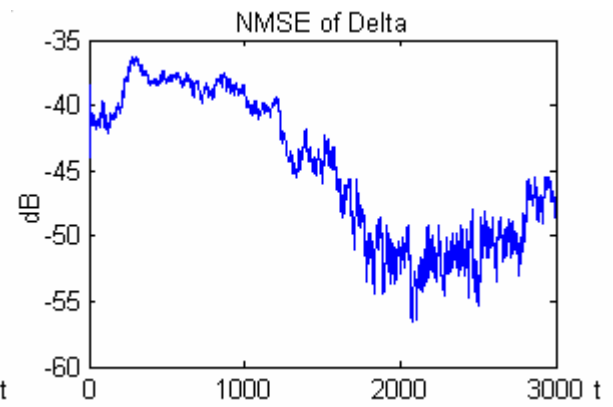
(g)



(h)



(i)



(j)

Fig. 4. (continued) g) Estimation of the skewness parameter, h) NMSE curve of the skewness parameter estimation, i) Estimation of the location parameter, j) NMSE curve of the location parameter estimation

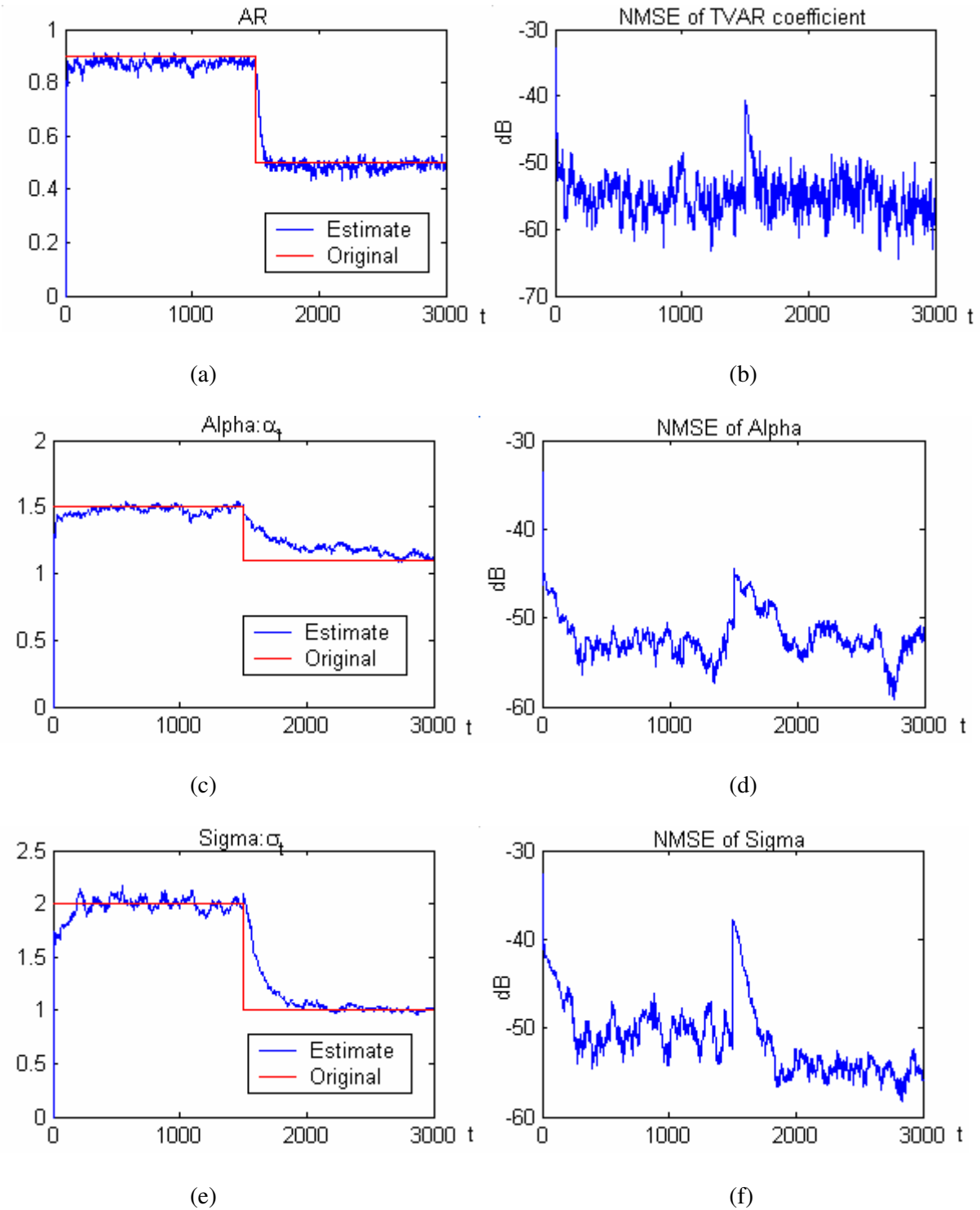
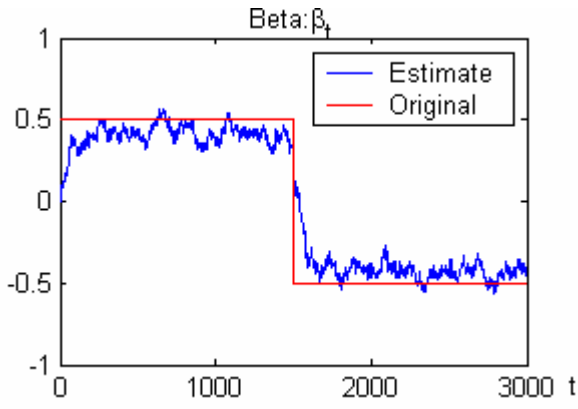
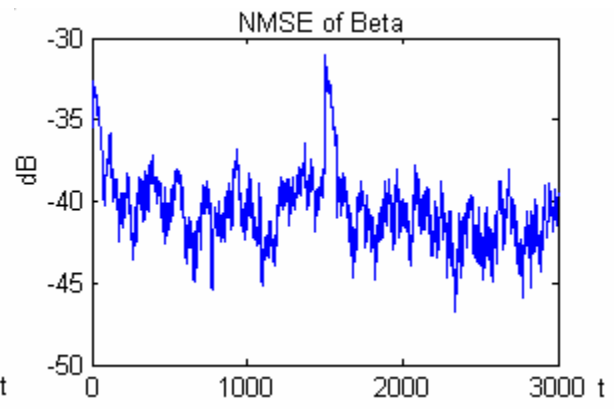


Fig. 5. Experiment B3: Estimation of time-varying AR coefficient and distribution parameters of *Skewed α -stable* process

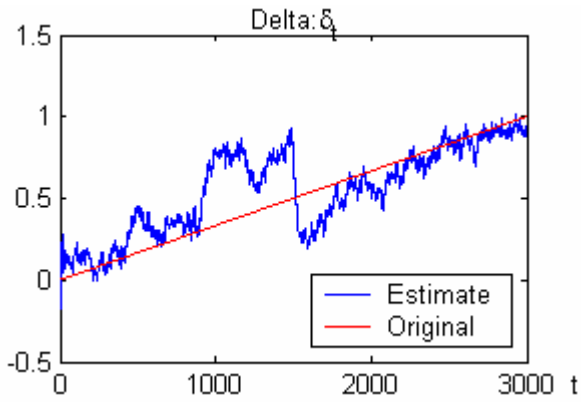
- a) Estimation of TVAR coefficient, b) NMSE curve of TVAR estimate, c) Estimation of the shape parameter, d) NMSE curve of the shape parameter estimation, e) Estimation of the dispersion parameter, f) NMSE curve of the dispersion parameter estimation



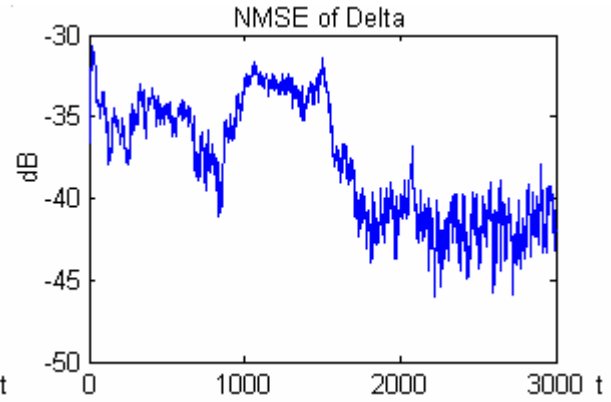
(g)



(h)



(i)



(j)

Fig. 5. (continued) g) Estimation of the skewness parameter, h) NMSE curve of the skewness parameter estimation, i) Estimation of the location parameter, j) NMSE curve of the location parameter estimation

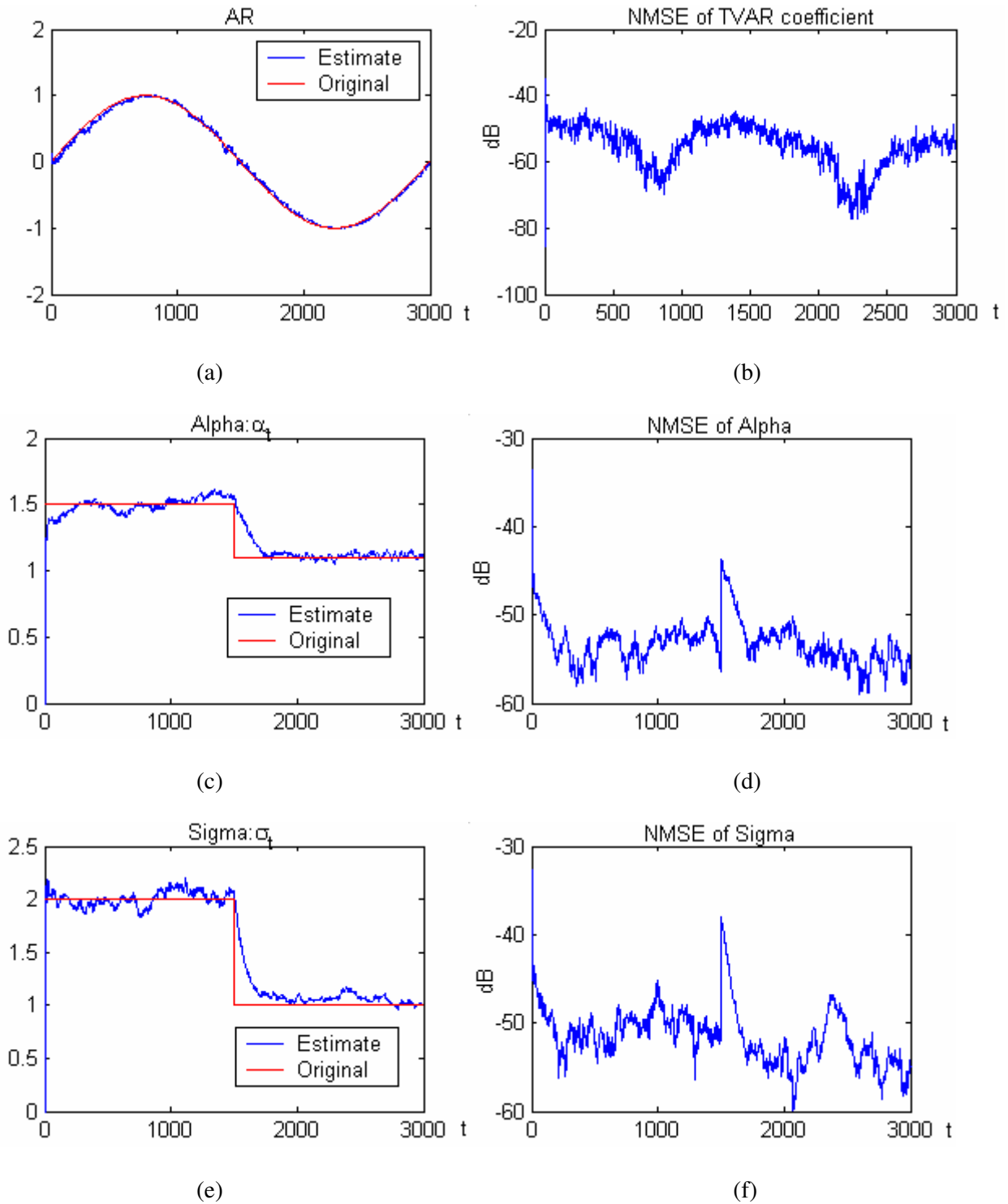


Fig. 6. Experiment C1: Estimation of time-varying AR coefficient and distribution parameters of $S\alpha S$ process

a) Estimation of TVAR coefficient, b) NMSE curve of TVAR estimate, c) Estimation of the shape parameter, d) NMSE curve of the shape parameter estimation, e) Estimation of the dispersion parameter, f) NMSE curve of the dispersion parameter estimation

LIST OF FIGURE CAPTIONS

Fig. 1. Experiment A1: Estimation of time-varying AR coefficient and distribution parameters of *SaS* process

a) Estimation of TVAR coefficient, b) NMSE curve of TVAR estimate, c) Estimation of the shape parameter, d) NMSE curve of the shape parameter estimation, e) Estimation of the dispersion parameter, f) NMSE curve of the dispersion parameter estimation

Fig. 2. Experiment A2: Estimation of time-varying AR coefficient and distribution parameters of *SaS* process

a) Estimation of TVAR coefficient, b) NMSE curve of TVAR estimate, c) Estimation of the shape parameter, d) NMSE curve of the shape parameter estimation, e) Estimation of the dispersion parameter, f) NMSE curve of the dispersion parameter estimation

Fig. 3. Experiment B1: Estimation of time-varying AR coefficient and distribution parameters of *Skewed α -stable* process , a)

Estimation of TVAR coefficient, b) NMSE curve of TVAR estimate, c) Estimation of the shape parameter, d) NMSE curve of the shape parameter estimation, e) Estimation of the dispersion parameter, f) NMSE curve of the dispersion parameter estimation, g) Estimation of the skewness parameter, h) NMSE curve of the skewness parameter estimation

Fig. 4. Experiment B2: Estimation of time-varying AR coefficient and distribution parameters of *Skewed α -stable* process, a)

Estimation of TVAR coefficient, b) NMSE curve of TVAR estimate, c) Estimation of the shape parameter, d) NMSE curve of the shape parameter estimation, e) Estimation of the dispersion parameter, f) NMSE curve of the dispersion parameter estimation, g) Estimation of the skewness parameter, h) NMSE curve of the skewness parameter estimation, i) Estimation of the location parameter, j) NMSE curve of the location parameter estimation

Fig. 5. Experiment B3: Estimation of time-varying AR coefficient and distribution parameters of *Skewed α -stable* process, a)

Estimation of TVAR coefficient, b) NMSE curve of TVAR estimate, c) Estimation of the shape parameter, d) NMSE curve of the shape parameter estimation, e) Estimation of the dispersion parameter, f) NMSE curve of the dispersion parameter estimation, g) Estimation of the skewness parameter, h) NMSE curve of the skewness parameter estimation, i) Estimation of the location parameter, j) NMSE curve of the location parameter estimation

Fig. 6. Experiment C1: Estimation of time-varying AR coefficient and distribution parameters of *SaS* process

a) Estimation of TVAR coefficient, b) NMSE curve of TVAR estimate, c) Estimation of the shape parameter, d) NMSE curve of the shape parameter estimation, e) Estimation of the dispersion parameter, f) NMSE curve of the dispersion parameter estimation

TABLE 1: Experimental Scenarios