Managing Uncertainty and Vagueness in Semantic Web Languages

Tutorial at SWAP-2007

Umberto Straccia

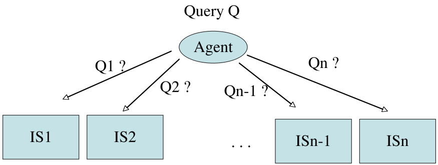
ISTI-CNR, Pisa, Italy
straccia@isti.cnr.it

Sources of Uncertainty and Vagueness on the Web

- Information Retrieval:
  - To which degree is a Web site, a Web page, a text passage, an image region, a video segment, . . . relevant to my information need?
- Matchmaking
  - To which degree does an object match my requirements?
    - if I'm looking for a car and my budget is *about* 20.000 €, to which degree does a car's price of 20.500 € match my budget?

- Semantic annotation
  - To which degree does e.g., an image object represent a dog?
- Information extraction
  - To which degree am I'm sure that e.g., SW is an acronym of "Semantic Web"?
- Ontology alignment (schema mapping)
  - To which degree do two concepts of two ontologies represent the same, or are disjoint, or are overlapping?
- Representation of background knowledge
  - To some degree birds fly.
  - To some degree Jim is a blond and young.

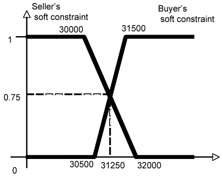
Example (Distributed Information Retrieval) [7]



Then the agent has to perform automatically the following steps:

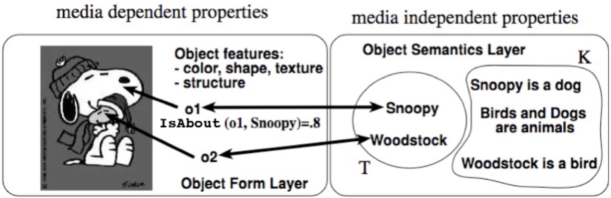
1. The agent has to select a subset of relevant resources $\mathscr{S}' \subseteq \mathscr{S}$, as it is not reasonable to assume to access to and query all resources (resource selection/resource discovery);
2. For every selected source $\mathcal{S}_i \in \mathscr{S}'$ the agent has to reformulate its information need Q_A into the query language $\mathcal{L}_i$ provided by the resource (schema mapping/ontology alignment);
3. The results from the selected resources have to be merged together (data fusion/rank aggregation)

Example (Negotiation) [2]



- A car seller sells an Audi TT for 31500 €, as from the catalog price.
- A buyer is looking for a sports-car, but wants to to pay not more than around 30000 €
- Classical DLs: the problem relies on the crisp conditions on price.
- More fine grained approach: to consider prices as vague constraints (fuzzy sets) (as usual in negotiation)
  - Seller would sell above 31500 €, but can go down to 30500 €
  - The buyer prefers to spend less than 30000 €, but can go up to 32000 €
  - Highest degree of matching is 0.75 . The car may be sold at 31250 €.

Example (Logic-based information retrieval model)[1, 8]



IsAbout		
ImageRegion	*Object ID*	*degree*
*o*1	*snoopy*	0.8
*o*2	*woodstock*	0.7
⋮	⋮	

"Find top-*k* image regions about animals"

$$Query(x) \leftarrow ImageRegion(x) \wedge isAbout(x, y) \wedge Animal(y)$$

Example (Database query) [3, 4, 5, 6]

HotelID	*hasLoc*
*h*1	*hl*1
*h*2	*hl*2
⋮	⋮

ConferenceID	*hasLoc*
*c*1	*cl*1
*c*2	*cl*2
⋮	⋮

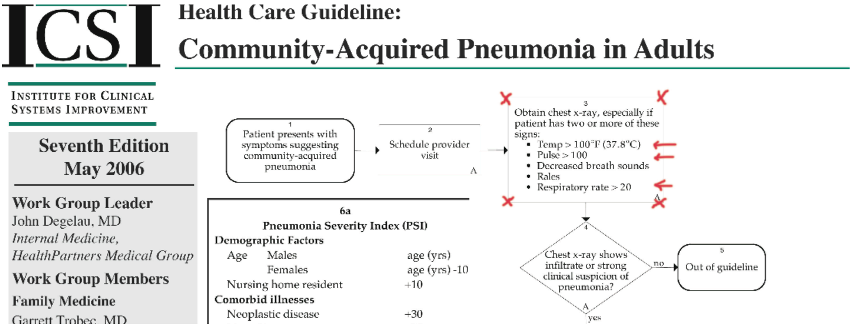
hasLoc	*hasLoc*	*distance*
*hl*1	*cl*1	300
*hl*1	*cl*2	500
*hl*2	*cl*1	750
*hl*2	*cl*2	800
⋮	⋮	

hasLoc	*hasLoc*	*close*	*cheap*
*hl*1	*cl*1	0.7	0.3
*hl*1	*cl*2	0.5	0.5
*hl*2	*cl*1	0.25	0.8
*hl*2	*cl*2	0.2	0.9
⋮	⋮	⋮	

“Find top-*k* cheapest hotels close to the train station”

$$q(h) \leftarrow hasLocation(h, hl) \wedge hasLocation(\texttt{train}, cl) \wedge close(hl, cl) \wedge cheap(h)$$

Example (Health-care: diagnosis of pneumonia)



- E.g., *Temp* = 37.5, *Pulse* = 98, *RespiratoryRate* = 18 are in the "danger zone" already
- Temperature, Pulse and Respiratory rate, . . . : these constraints are rather imprecise than crisp

Uncertainty vs. Vagueness: a clarification

- What does the degree mean?
- There is often a misunderstanding between interpreting a degree as a measure of uncertainty or as a measure of vagueness
- The value 0.83 has a different interpretation in “Birds fly to degree 0.83” from that in “Hotel Verdi is close to the train station to degree 0.83”

Uncertainty

- Uncertainty: statements are true or false. But, due to lack of knowledge we can only estimate to which probability/possibility/necessity degree they are true or false
  - For instance, a bird flies or does not fly. The probability/possibility/necessity degree that it flies is 0.83
- Usually we have a possible world semantics with a distribution over possible worlds:

$$W = \{I \text{ classical interpretation}\}, \quad I(\varphi) \in \{0, 1\}$$

$$\mu \colon W \to [0, 1], \quad \mu(I) \in [0, 1]$$

$$Pr(\phi) = \sum_{I \models \phi} \mu(I)$$

$$Poss(\phi) = \sup_{I \models \phi} \mu(I)$$

$$Necc(\phi) = \inf_{I \not\models \phi} \mu(I) = 1 - Poss(\neg \phi)$$

Vagueness

- Vagueness: statements involve concepts for which there is no exact definition, such as tall, small, close, far, cheap, expensive, isAbout, similarTo. Statements are true to some degree which is taken from a truth space.
  - E.g., “Hotel Verdi is close to the train station to degree 0.83”
- Truth space: set of truth values L and an partial order $\leq$
- Many-valued Interpretation: a function I mapping formulae into L, i.e. $I(\varphi) \in L$
- Fuzzy Logic: $L = [0, 1]$
- Uncertainty and Vagueness: “It is possible/probable to degree 0.83 that it will be hot tomorrow”
- The notion of imperfect information covers concepts such as uncertainty, vagueness, contradiction, incompleteness, imprecision.

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Web Ontology Languages

- Wide variety of languages for “Explicit Specification”
  - Graphical notations
    - Semantic networks
    - UML
    - RDF/RDFS
  - Logic based
    - Description Logics (e.g., OIL, DAML+OIL, OWL, OWL-DL, OWL-Lite)
    - Rules (e.g., RuleML, RIF, SWRL, LP/Prolog)
    - First Order Logic (e.g., KIF)
- RDF and OWL-DL are the major players (so far ...)

RDF

- Statements are of the form

$$\langle \textit{subject}, \textit{predicate}, \textit{object} \rangle$$

  called triples: e.g.

$$\langle \textit{umberto}, \textit{plays}, \textit{soccer} \rangle$$

- can be represented graphically as:

$$\boxed{\text{umberto}} \xrightarrow{\textit{plays}} \boxed{\text{soccer}}$$

- Statements describe properties of resources
- A resource is any object that can be pointed to by a URI (Universal Resource Identifier):

RDF Schema (RDFS)

- RDF Schema allows you to define vocabulary terms and the relations between those terms
- RDF Schema terms (just a few examples):
  - Class
  - Property
  - type
  - subClassOf
  - range
  - domain
- These terms are the RDF Schema building blocks (constructors) used to create vocabularies:

```
<Person,type, Class>
<hasColleague, type, Property>
<Professor, subClassOf,Person>
<Carole, type,Professor>
<hasColleague, range,Person>
<hasColleague, domain,Person>
```

OWL [10]

- Three species of OWL
  - OWL full is union of OWL syntax and RDF (Undecidable)
  - OWL DL restricted to FOL fragment (decidable in NEXPTIME)
  - OWL Lite is “easier to implement” subset of OWL DL (decidable in EXPTIME)
- Semantic layering
  - OWL DL within Description Logic (DL) fragment
- OWL DL based on $\mathcal{SHOIN}(D_n)$ DL
- OWL Lite based on $\mathcal{SHIF}(D_n)$ DL

Description Logics (DLs)

- The logics behind OWL-DL and OWL-Lite, `http://dl.kr.org/`.
- Concept/Class: names are equivalent to unary predicates
  - In general, concepts equiv to formulae with one free variable
- Role or attribute: names are equivalent to binary predicates
  - In general, roles equiv to formulae with two free variables
- Taxonomy: Concept and role hierarchies can be expressed
- Individual: names are equivalent to constants
- Operators: restricted so that:
  - Language is decidable and, if possible, of low complexity
  - No need for explicit use of variables
    - Restricted form of $\exists$ and $\forall$
  - Features such as counting can be succinctly expressed

The DL Family

- A given DL is defined by set of concept and role forming operators
- Basic language: $\mathcal{ALC}$($\mathcal{A}$ttributive $\mathcal{L}$anguage with $\mathcal{C}$omplement)

Syntax				Semantics	Example
C, D	$\to$	$\top$	$\mid$	$\top(x)$	
		$\bot$	$\mid$	$\bot(x)$	
		A	$\mid$	$A(x)$	*Human*
		$C \sqcap D$	$\mid$	$C(x) \wedge D(x)$	$Human \sqcap Male$
		$C \sqcup D$	$\mid$	$C(x) \vee D(x)$	$Nice \sqcup Rich$
		$\neg C$	$\mid$	$\neg C(x)$	$\neg Meat$
		$\exists R.C$	$\mid$	$\exists y.R(x,y) \wedge C(y)$	$\exists has\_child.Blond$
		$\forall R.C$		$\forall y.R(x,y) \Rightarrow C(y)$	$\forall has\_child.Human$
$C \sqsubseteq D$				$\forall x.C(x) \Rightarrow D(x)$	$Happy\_Father \sqsubseteq Man \sqcap \exists has\_child.Female$
$a{:}C$				$C(a)$	$John{:}Happy\_Father$

Toy Example

$$\begin{array}{rcl} Sex & = & Male \sqcup Female \\ Male \sqcap Female & \sqsubseteq & \bot \\ Person & \sqsubseteq & Human \sqcap \exists hasSex.Sex \\ MalePerson & \sqsubseteq & Person \sqcap \exists hasSex.Male \end{array}$$

$$umberto{:}Person \sqcap \exists hasSex.\neg Female$$

$$KB \models umberto{:}MalePerson$$

Note on DL Naming

- $\mathcal{AL}$: $C, D \longrightarrow \top \mid \bot \mid A \mid C \sqcap D \mid \neg A \mid \exists R.\top \mid \forall R.C$
- $\mathcal{C}$: Concept negation, $\neg C$. Thus, $\mathcal{ALC} = \mathcal{AL} + \mathcal{C}$
- $\mathcal{S}$: Used for $\mathcal{ALC}$ with transitive roles $\mathcal{R}_+$
- $\mathcal{U}$: Concept disjunction, $C_1 \sqcup C_2$
- $\mathcal{E}$: Existential quantification, $\exists R.C$
- $\mathcal{H}$: Role inclusion axioms, $R_1 \sqsubseteq R_2$, e.g. $is\_component\_of \sqsubseteq is\_part\_of$
- $\mathcal{N}$: Number restrictions, $(\geq n\ R)$ and $(\leq n\ R)$, e.g. $(\geq 3\ has\_Child)$ (has at least 3 children)
- $\mathcal{Q}$: Qualified number restrictions, $(\geq n\ R.C)$ and $(\leq n\ R.C)$, e.g. $(\leq 2\ has\_Child.Adult)$ (has at most 2 adult children)
- $\mathcal{O}$: Nominals (singleton class), $\{a\}$, e.g. $\exists has\_child.\{mary\}$.
  Note: $a{:}C$ equiv to $\{a\} \sqsubseteq C$ and $(a, b){:}R$ equiv to $\{a\} \sqsubseteq \exists R.\{b\}$
- $\mathcal{I}$: Inverse role, R^-, e.g. $isPartOf = hasPart^-$
- $\mathcal{F}$: Functional role, f, e.g. $functional(hasAge)$
- $\mathcal{R}_+$: transitive role, e.g. $transitive(isPartOf)$

For instance,

$$\mathcal{SHIF} = \mathcal{S} + \mathcal{H} + \mathcal{I} + \mathcal{F} = \mathcal{ALCR_+HIF}$$ OWL-Lite (EXPTIME)

$$\mathcal{SHOIN} = \mathcal{S} + \mathcal{H} + \mathcal{O} + \mathcal{I} + \mathcal{N} = \mathcal{ALCR_+HOIN}$$ OWL-DL (NEXPTIME)

Concrete Domains

- Concrete domains: reals, integers, strings, ...

$$(tim, 14){:}hasAge$$
$$(sf, \text{``}SoftComputing\text{''}){:}hasAcronym$$
$$(source1, \text{``}ComputerScience\text{''}){:}isAbout$$
$$(service2, \text{``}InformationRetrievalTool''){:}Matches$$
$$Minor = Person \sqcap \exists hasAge. \leq_{18}$$

- Semantics: a clean separation between "object" classes and concrete domains
  - $D = \langle \Delta_D, \Phi_D \rangle$
  - Δ_D is an interpretation domain
  - Φ_D is the set of concrete domain predicates d with a predefined arity n and fixed interpretation $d^D \subseteq \Delta_D^n$
  - Concrete properties: $R^\mathcal{I} \subseteq \Delta^\mathcal{I} \times \Delta_D$
- Notation: (D). E.g., $\mathcal{ALC}(D)$ is $\mathcal{ALC}$ + concrete domains

LPs Basics (for ease, without default negation) [6]

- Predicates are n-ary
- Terms are variables or constants
- Rules are of the form

$$P(\mathbf{x}) \leftarrow \varphi(\mathbf{x}, \mathbf{y})$$

where $\varphi(\mathbf{x}, \mathbf{y})$ is a formula built from atoms of the form $B(\mathbf{z})$ and connectors $\wedge, \vee$
For instance,

$$has\_father(x, y) \quad \leftarrow \quad has\_parent(x, y) \wedge Male(y)$$

- Facts are rules with empty body
For instance,

$$has\_parent(mary, jo)$$

Toy Example

$$
\begin{array}{rcl}
Q(x) & \leftarrow & B(x) \\
Q(x) & \leftarrow & C(x) \\
B(a) & \leftarrow & \\
C(b) & \leftarrow &
\end{array}
$$

$$KB \models Q(a) \quad KB \models Q(b) \qquad answers(KB, Q) = \{a, b\}$$

where $answers(KB, Q) = \{\mathbf{c} \mid KB \models Q(\mathbf{c})\}$

DLPs Basics

- Combine DLs with LPs:
  - DL atoms and roles may appear in rules

$$\begin{array}{lcl} buy(x) & \leftarrow & Electronics(x), offer(x) \\ Camera & \sqsubseteq & Electronics \end{array}$$

- Knowledge Base is a pair $KB = \langle \mathcal{P}, \Sigma \rangle$, where
  - $\mathcal{P}$ is a logic program
  - Σ is a DL knowledge base (set of assertions and inclusion axioms)
- Many different approaches exists with different semantics

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Probabilistic Logic

- Integration of (propositional) logic- and probability-based representation and reasoning formalisms.
- Reasoning from logical constraints and interval restrictions for conditional probabilities (also called *conditional constraints*).
- Reasoning from convex sets of probability distributions.
- Model-theoretic notion of logical entailment.

Syntax of Probabilistic Knowledge Bases

- Finite nonempty set of basic events $\Phi = \{p_1, \ldots, p_n\}$.
- Event ϕ: Boolean combination of basic events
- Logical constraint $\psi \Leftarrow \phi$: events ψ and ϕ: "ϕ implies ψ".
- Conditional constraint $(\psi|\phi)[l, u]$: events ψ and ϕ, and $l, u \in [0, 1]$: "conditional probability of ψ given ϕ is in $[l, u]$".
- Probabilistic knowledge base $KB = (L, P)$:
  - finite set of logical constraints L,
  - finite set of conditional constraints P.

Example

Probabilistic knowledge base $KB = (L, P)$:

- $L = \{bird \Leftarrow eagle\}$:

  "All eagles are birds".

- $P = \{(have\_legs \mid bird)[1, 1], (fly \mid bird)[0.95, 1]\}$:

  "All birds have legs".

  "Birds fly with a probability of at least 0.95".

Semantics of Probabilistic Knowledge Bases

- World I: truth assignment to all basic events in Φ.
- $\mathcal{I}_\Phi$: all worlds for Φ.
- Probabilistic interpretation Pr: probability function on $\mathcal{I}_\Phi$.
- $Pr(\phi)$: sum of all $Pr(I)$ such that $I \in \mathcal{I}_\Phi$ and $I \models \phi$.
- $Pr(\psi|\phi)$: if $Pr(\phi) > 0$, then $Pr(\psi|\phi) = Pr(\psi \wedge \phi) \,/\, Pr(\phi)$.
- Truth under Pr:
  - $Pr \models \psi \Leftarrow \phi$ iff $Pr(\psi \wedge \phi) = Pr(\phi)$
    (iff $Pr(\psi \Leftarrow \phi) = 1$).
  - $Pr \models (\psi|\phi)[l, u]$ iff $Pr(\psi \wedge \phi) \in [l, u] \cdot Pr(\phi)$
    (iff either $Pr(\phi) = 0$ or $Pr(\psi|\phi) \in [l, u]$).

Example

- Set of basic propositions $\Phi = \{bird, fly\}$.
- $\mathcal{I}_\Phi$ contains exactly the worlds I_1, I_2, I_3, and I_4 over Φ:

	fly	$\neg$*fly*
bird	I_1	I_2
$\neg$*bird*	I_3	I_4

- Some probabilistic interpretations:

Pr_1	*fly*	$\neg$*fly*
bird	19/40	1/40
$\neg$*bird*	10/40	10/40

Pr_2	*fly*	$\neg$*fly*
bird	0	1/3
$\neg$*bird*	1/3	1/3

- $Pr_1(fly \wedge bird) = 19/40$ and $Pr_1(bird) = 20/40$.
- $Pr_2(fly \wedge bird) = 0$ and $Pr_2(bird) = 1/3$.
- $\neg fly \Leftarrow bird$ is false in Pr_1, but true in Pr_2.
- $(fly \mid bird)[.95, 1]$ is true in Pr_1, but false in Pr_2.

Satisfiability and Logical Entailment

- Pr is a model of $KB = (L, P)$ iff $Pr \models F$ for all $F \in L \cup P$.
- KB is satisfiable iff a model of KB exists.
- $KB \| \!\!= (\psi|\phi)[l, u]$: $(\psi|\phi)[l, u]$ is a logical consequence of KB iff every model of KB is also a model of $(\psi|\phi)[l, u]$.
- $KB \| \!\!=_{tight} (\psi|\phi)[l, u]$: $(\psi|\phi)[l, u]$ is a tight logical consequence of KB iff l (resp., u) is the infimum (resp., supremum) of $Pr(\psi|\phi)$ subject to all models Pr of KB with $Pr(\phi) > 0$.

Example

- Probabilistic knowledge base:

$$KB = (\{bird \Leftarrow eagle\}, \{(have\_legs \mid bird)[1,1], (fly \mid bird)[0.95,1]\}).$$

- KB is satisfiable, since

  Pr with $Pr(bird \wedge eagle \wedge have\_legs \wedge fly) = 1$ is a model.

- Some conclusions under logical entailment:

  $KB \Vdash (have\_legs \mid bird)[0.3,1]$, $KB \Vdash (fly \mid bird)[0.6,1]$.

- Tight conclusions under logical entailment:

  $KB \Vdash_{tight} (have\_legs \mid bird)[1,1]$, $KB \Vdash_{tight} (fly \mid bird)[0.95,1]$,
  $KB \Vdash_{tight} (have\_legs \mid eagle)[1,1]$, $KB \Vdash_{tight} (fly \mid eagle)[0,1]$.

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Probabilistic Ontologies

Main types of encoded probabilistic knowledge:

- Terminological probabilistic knowledge about concepts and roles: “Birds fly with a probability of at least 0.95”.
- Assertional probabilistic knowledge about instances of concepts and roles: “Tweety is a bird with a probability of at least 0.9”.

Main types of reasoning problems:

- Satisfiability of the terminological probabilistic knowledge.
- Tight conclusions about generic objects (from the terminological probabilistic knowledge).
- Satisfiability of the assertional probabilistic knowledge.
- Tight conclusions about concrete objects (from both the terminological and the assertional probabilistic knowledge).

Use of Probabilistic Ontologies

- Representation of terminological and assertional probabilistic knowledge (e.g., in the medical domain or at the stock exchange market).
- Information retrieval, for an increased recall (e.g., Udrea et al.: Probabilistic ontologies and relational databases. In *Proc. CoopIS/DOA/ODBASE-2005*).
- Ontology matching (e.g., Mitra et al.: OMEN: A probabilistic ontology mapping tool. In *Proc. ISWC-2005*).
- Probabilistic data integration, especially for handling ambiguous and controversial pieces of information.

Probabilistic RDF

O. Udrea, V. S. Subrahmanian, and Z. Majkic. Probabilistic RDF. In *Proceedings IRI-2006*.

- probabilistic generalization of RDF
- terminological probabilistic knowledge about classes
- assertional probabilistic knowledge about properties of individuals
- assertional probabilistic inference for acyclic probabilistic RDF theories, which is based on logical entailment in probabilistic logic, coupled with a local probabilistic semantics

Probabilistic DLs

R. Giugno, T. Lukasiewicz. P-$\mathcal{SHOQ}(\mathbf{D})$: A probabilistic extension of $\mathcal{SHOQ}(\mathbf{D})$ for probabilistic ontologies in the SW. In *Proc. JELIA-2002*.

- probabilistic generalization of the description logic $\mathcal{SHOQ}(\mathbf{D})$ (recently also extended to $\mathcal{SHIF}(\mathbf{D})$ and $\mathcal{SHOIN}(\mathbf{D})$)
- terminological probabilistic knowledge about concepts and roles
- assertional probabilistic knowledge about instances of concepts and roles
- terminological probabilistic inference based on lexicographic entailment in probabilistic logic (stronger than logical entailment)
- assertional probabilistic inference based on lexicographic entailment in probabilistic logic (for combining assertional and terminological probabilistic knowledge)
- terminological and assertional probabilistic inference problems reduced to sequences of linear optimization problems

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Possibilistic DLs

Generalization of DLs by possibilistic uncertainty, which is based on possibilistic interpretations rather than probabilistic interpretations.

Possibilistic interpretation: mapping $\pi\colon \mathcal{I}_\Phi \to [0,1]$.

"$\pi(I)$ is the degree to which the world I is possible."

Poss(ϕ): possibility of ϕ in π: $Poss(\phi) = \max\{\pi(I) \mid I \in \mathcal{I}_\Phi,\ I \models \phi\}$

- B. Hollunder. An alternative proof method for possibilistic logic and its application to terminological logics. *Int. J. Approx. Reasoning*, 12(2):85–109, 1995.
- D. Dubois, J. Mengin, and H. Prade. Possibilistic uncertainty and fuzzy features in description logic: A preliminary discussion. In E. Sanchez, editor, *Capturing Intelligence: Fuzzy Logic and the Semantic Web*, 2006.
- C.-J. Liau and Y. Y. Yao. Information retrieval by possibilistic reasoning. In *Proc. DEXA-2001*.

Other Works

- Z. Ding and Y. Peng. A probabilistic extension to ontology language OWL. In *Proceedings HICSS-2004*.
- Y. Yang and J. Calmet. OntoBayes: An ontology-driven uncertainty model. In *Proceedings IAWTIC-2005*.
- Z. Ding, Y. Peng, and R. Pan. BayesOWL: Uncertainty modeling in Semantic Web ontologies. In Z. Ma, editor, *Soft Computing in Ontologies and Semantic Web*. Springer, 2006.
- H. Nottelmann and N. Fuhr. Adding probabilities and rules to OWL Lite subsets based on probabilistic Datalog. *IJUFKS*, 14(1):17–42, 2006.

Probabilistic Logic Programs

Probabilistic generalizations of logic programs / rule-based systems / deductive databases / Datalog:

(1) Probabilistic generalizations of (annotated) logic programs based on probabilistic logic (no uncertainty degrees associated with rules):

- R. T. Ng and V. S. Subrahmanian. Probabilistic logic programming. *Inf. Comput.*, 101(2):150–201, 1992.
- R. T. Ng and V. S. Subrahmanian. A semantical framework for supporting subjective and conditional probabilities in deductive databases. *J. Autom. Reasoning*, 10(2):191–235, 1993.
- A. Dekhtyar and V. S. Subrahmanian. Hybrid probabilistic programs. *J. Log. Program.* 43(3):187–250, 2000.

(2) Probabilistic generalizations of logic programs based on Bayesian networks / causal models:

- D. Poole. Probabilistic Horn abduction and Bayesian networks. *Artif. Intell.*, 64:81–129, 1993.
- D. Poole. The independent choice logic for modeling multiple agents under uncertainty. *Artif. Intell.*, 94:7–56, 1997.
- K. Kersting and L. De Raedt. Bayesian logic programs. *CoRR*, cs.AI/0111058, 2001.
- C. Baral, M. Gelfond, and J. N. Rushton. Probabilistic reasoning with answer sets. In *Proceedings LPNMR-2004*.

(3) Relational Bayesian networks:

- M. Jaeger. Relational Bayesian networks. In *Proc. UAI-1997*.
- D. Koller and A. Pfeffer. Object-oriented Bayesian networks. In *Proceedings UAI-1997*.
- H. Pasula and S. J. Russell. Approximate inference for first-order probabilistic languages. In *Proceedings IJCAI-2001*.
- D. Poole. First-order probabilistic inference. In *Proc. IJCAI-2003*.

(4) First-order generalization of probabilistic knowledge bases in probabilistic logic (based on logical entailment, lexicographic entailment, and maximum entropy entailment):

- T. Lukasiewicz. Probabilistic logic programming. In *Proceedings ECAI-1998*.
- T. Lukasiewicz. Probabilistic logic programming with conditional constraints. *ACM TOCL* 2(3):289–339, 2001.
- T. Lukasiewicz. Probabilistic logic programming under inheritance with overriding. In *Proceedings UAI-2001*.
- G. Kern-Isberner and T. Lukasiewicz. Combining probabilistic logic programming with the power of maximum entropy. *Artif. Intell.*, 157(1–2):139–202, 2004.

Probabilistic Description Logic Programs

T. Lukasiewicz. Probabilistic description logic programs. *IJAR*, 2007.

- Probabilistic dl-programs generalize (loosely coupled) dl-programs by probabilistic uncertainty as in Poole's ICL.
- They properly generalize Poole's ICL.
- They consist of a dl-program along with a probability distribution μ over total choices *B*.
- They specify a set of distributions over first-order models: Every total choice *B* along with the dl-program specifies a set of first-order models of which the probabilities should sum up to $\mu(B)$.
- There are also tightly coupled probabilistic dl-programs.
- Important applications are data integration and ontology mapping under probabilistic uncertainty and inconsistency.

Example

Description logic knowledge base L
of a probabilistic dl-program $KB = (L, P, C, \mu)$:

$PC \sqcup Camera \sqsubseteq Electronics$; $PC \sqcap Camera \sqsubseteq \bot$;
$Book \sqcup Electronics \sqsubseteq Product$; $Book \sqcap Electronics \sqsubseteq \bot$;
$Textbook \sqsubseteq Book$;

$Product \sqsubseteq\ \geq 1\ related$;
$\geq 1\ related \sqcup \geq 1\ related^{-} \sqsubseteq Product$;

$Textbook(tb\_ai)$; $Textbook(tb\_lp)$;
$PC(pc\_ibm)$; $PC(pc\_hp)$;

$related(tb\_ai, tb\_lp)$; $related(pc\_ibm, pc\_hp)$;
$provides(ibm, pc\_ibm)$; $provides(hp, pc\_hp)$.

Classical dl-rules in P
of a probabilistic dl-program $KB = (L, P, C, \mu)$:

- $pc(pc\_1)$; $pc(pc\_2)$; $pc(pc\_3)$;
- $brand\_new(pc\_1)$; $brand\_new(pc\_2)$;
- $vendor(dell, pc\_1)$; $vendor(dell, pc\_2)$; $vendor(dell, pc\_3)$;
- $provider(P) \leftarrow vendor(P, X), DL[PC \uplus pc; Product](X)$;
- $provider(P) \leftarrow DL[provides](P, X), DL[PC \uplus pc; Product](X)$;
- $similar(X, Y) \leftarrow DL[related](X, Y)$;
- $similar(X, Z) \leftarrow similar(X, Y), similar(Y, Z)$.

Probabilistic dl-rules in P along with the probability μ on the choice space C of a probabilistic dl-program $KB = (L, P, C, \mu)$:

- $avoid(X) \leftarrow DL[Camera](X), \text{not } offer(X), avoid\_pos;$
- $offer(X) \leftarrow DL[PC \uplus pc; Electronics](X), \text{not } brand\_new(X), offer\_pos;$
- $buy(C, X) \leftarrow needs(C, X), view(X), \text{not } avoid(X), v\_buy\_pos;$
- $buy(C, X) \leftarrow needs(C, X), buy(C, Y), also\_buy(Y, X), a\_buy\_pos.$

μ: $avoid\_pos, avoid\_neg \mapsto 0.9, 0.1;$ $offer\_pos, offer\_neg \mapsto 0.9, 0.1;$ $v\_buy\_pos, v\_buy\_neg \mapsto 0.7, 0.3;$ $a\_buy\_pos, a\_buy\_neg \mapsto 0.7, 0.3.$

$\{avoid\_pos, offer\_pos, v\_buy\_pos, a\_buy\_pos\} : 0.9 \times 0.9 \times 0.7 \times 0.7, \ldots$

Probabilistic query: $\exists(buy(c, x) \mid needs(c, x) \wedge buy(c, y) \wedge also\_buy(y, x) \wedge view(x) \wedge \neg avoid(x))[L, U]$

Example: Probabilistic Data Integration

Obtain a weather forecast by integrating the potentially different weather forecasts of three weather forecast institutes A, B, and C.

Our trust in the institutes A, B, and C is expressed by the trust probabilities 0.6, 0.3, and 0.1, respectively.

Probabilistic integration of the source schemas of A, B, and C to the global schema G is specified by the following $KB_M = (\emptyset, P_M, C_M, \mu_M)$:

$$\begin{aligned} P_M = \{ & forecast\_rome(D, W, T, M) \leftarrow forecast(rome, D, W, T, M),\ inst_A; \\ & forecast\_rome(D, W, T, M) \leftarrow forecastRome(D, W, T, M),\ inst_B; \\ & forecast\_rome(D, W, T, M) \leftarrow forecast\_weather(rome, D, W), \\ & \quad forecast\_temperature(rome, D, T), \\ & \quad forecast\_wind(rome, D, M),\ inst_C \}\,; \end{aligned}$$

$$C_M = \{\{inst_A, inst_B, inst_C\}\}\,;$$

$$\mu_M : \ inst_A, inst_B, inst_C \mapsto 0.6, 0.3, 0.1\,.$$

Example (Tightly Coupled): Ontology Mapping

The global schema contains the concept *logic_programming*, while the source schemas contain only the concepts *rule-based_systems* resp. *deductive_databases* in their ontologies.

A randomly chosen book from the area *rule-based_systems* (resp., *deductive_databases*) may belong to *logic_programming* with the probability 0.7 (resp., 0.8).

Probabilistic mapping from the two source schemas to the global schema expressed by the following $KB_M = (\emptyset, P_M, C_M, \mu_M)$:

$$
\begin{aligned}
P_M = \{ & \mathit{logic\_programming}(X) \leftarrow \mathit{rule\text{-}based\_systems}(X),\ \mathit{choice}_1\,;\\
& \mathit{logic\_programming}(X) \leftarrow \mathit{deductive\_databases}(X),\ \mathit{choice}_2\}\,;\\
C_M = \{ & \{\mathit{choice}_1, \mathit{not\_choice}_1\}, \{\mathit{choice}_2, \mathit{not\_choice}_2\}\}\,;\\
\mu_M :\ & \mathit{choice}_1, \mathit{not\_choice}_1, \mathit{choice}_2, \mathit{not\_choice}_2 \ \mapsto\ 0.7,\ 0.3,\ 0.8,\ 0.2\,.
\end{aligned}
$$

Vagueness

- Vagueness: statements involve concepts for which there is no exact definition, such as tall, close, cheap, IsAbout, simialarTo ...
- Statements are true to some degree which is taken from a truth space
    - E.g., "Hotel Verdi is close to the train station to degree 0.83"
    - "Find top-k cheapest hotels close to the train station"

$$q(h) \leftarrow hasLocation(h, hl) \wedge hasLocation(\texttt{train}, cl) \wedge close(hl, cl) \wedge cheap(h)$$

- Truth space: usually $[0, 1]$
- Interpretation: a function I mapping atoms into $[0, 1]$, i.e. $I(A) \in [0, 1]$
- Problem: what is the interpretation of e.g. $close(verdi, train) \wedge cheap(200)$?
    - E.g., if $I(close(verdi, train)) = 0.83$ and $I(cheap(200)) = 0.2$, what is the result of $0.83 \wedge 0.2$?
- More generally, what is the result of $n \wedge m$, for $n, m \in [0, 1]$?
- The choice cannot be any arbitrary computable function, but has to reflect some basic properties that one expects to hold for a "conjunction"

Propositional Fuzzy Logics Basics [5]

- Formulae: propositional formulae
- Truth space is $[0, 1]$
- Formulae have a a degree of truth in $[0, 1]$
- Interpretation: is a mapping $\mathcal{I} : \mathit{Atoms} \to [0, 1]$
- Interpretations are extended to formulae using norms to interpret connectives $\wedge, \vee, \neg, \to$

Typical norms

	Lukasiewicz Logic	Gödel Logic	Product Logic	Zadeh
$\neg x$	$1 - x$	if $x = 0$ then 1 else 0	if $x = 0$ then 1 else 0	$1 - x$
$x \wedge y$	$\max(x + y - 1, 0)$	$\min(x, y)$	$x \cdot y$	$\min(x, y)$
$x \vee y$	$\min(x + y, 1)$	$\max(x, y)$	$x + y - x \cdot y$	$\max(x, y)$
$x \Rightarrow y$	if $x \leq y$ then 1 else $1 - x + y$	if $x \leq y$ then 1 else y	if $x \leq y$ then 1 else y/x	$\max(1 - x, y)$

Note: for Lukasiewicz Logic and Zadeh, $x \Rightarrow y \equiv \neg x \vee y$

$$
\begin{array}{rcl}
\mathcal{I}(\phi \wedge \psi) & = & \mathcal{I}(\phi) \wedge \mathcal{I}(\psi) \\
\mathcal{I}(\phi \vee \psi) & = & \mathcal{I}(\phi) \vee \mathcal{I}(\psi) \\
\\
\mathcal{I}(\phi \rightarrow \psi) & = & \mathcal{I}(\phi) \rightarrow \mathcal{I}(\psi) \\
\mathcal{I} \models \phi & \text{iff} & \mathcal{I}(\phi) = 1 \quad \text{iff } \phi \text{ satisfiable} \\
\mathcal{I} \models \mathcal{T} & \text{iff} & \mathcal{I} \models \phi \text{ for all } \phi \in \mathcal{T} \\
\models \phi & \text{iff} & \text{for all } \mathcal{I} . \mathcal{I} \models \phi \\
\mathcal{T} \models \phi & \text{iff} & \text{for all } \mathcal{I}. \text{ if } \mathcal{I} \models \mathcal{T} \text{ then } \mathcal{I} \models \phi
\end{array}
$$

- Note:

$$
\begin{array}{rcl}
\neg\phi & \text{is} & \phi \to 0 \\
\phi\bar{\wedge}\psi & \text{defined as} & \phi \wedge (\phi \to \psi) \\
\phi\bar{\vee}\psi & \text{defined as} & ((\phi \to \psi) \to \psi)\bar{\wedge}((\psi \to \phi) \to \phi) \\
\mathcal{I}(\phi\bar{\wedge}\psi) & = & \min(\mathcal{I}(\phi), \mathcal{I}(\psi)) \\
\mathcal{I}(\phi\bar{\vee}\psi) & = & \max(\mathcal{I}(\phi), \mathcal{I}(\psi))
\end{array}
$$

- Zadeh semantics: not interesting for fuzzy logicians: its a sub-logic of Łukasiewicz and, thus, rarely considered by fuzzy logicians

$$
\begin{array}{rcl}
\neg_Z \phi & = & \neg_Ł \phi \\
\phi \wedge_Z \psi & = & \phi \wedge_Ł (\phi \to_Ł \psi) \\
\phi \to_Z \psi & = & \neg_Ł \phi \vee_Ł \psi
\end{array}
$$

Some additional properties of t-norms, s-norms, implication functions, and negation functions of various fuzzy logics.

Property	Łukasiewicz Logic	Gödel Logic	Product Logic	Zadeh Logic
$x \wedge \neg x = 0$	•	•	•	
$x \vee \neg x = 1$	•			
$x \wedge x = x$		•		•
$x \vee x = x$		•		•
$\neg \neg x = x$	•			•
$x \rightarrow y = \neg x \vee y$	•			•
$\neg (x \rightarrow y) = x \wedge \neg y$	•			•
$\neg (x \wedge y) = \neg x \vee \neg y$	•	•	•	•
$\neg (x \vee y) = \neg x \wedge \neg y$	•	•	•	•

Predicate Fuzzy Logics Basics [5]

- Formulae: First-Order Logic formulae, *terms* are either variables or constants
  - we may introduce functions symbols as well, with crisp semantics (but uninteresting), or we need to discuss also fuzzy equality (which we leave out here)
- Truth space is [0, 1]
- Formulae have a a degree of truth in [0, 1]
- Interpretation: is a mapping $\mathcal{I} : \mathit{Atoms} \to [0, 1]$
- Interpretations are extended to formulae as follows:

$$\begin{array}{rcl} \mathcal{I}(\neg\phi) & = & \mathcal{I}(\phi) \to 0 \\ \mathcal{I}(\phi \wedge \psi) & = & \mathcal{I}(\phi) \wedge \mathcal{I}(\psi) \\ \mathcal{I}(\phi \to \psi) & = & \mathcal{I}(\phi) \to \mathcal{I}(\psi) \\ \mathcal{I}(\exists x\phi) & = & \sup_{c \in \Delta^{\mathcal{I}}} \mathcal{I}^c_x(\phi) \\ \mathcal{I}(\forall x\phi) & = & \inf_{c \in \Delta^{\mathcal{I}}} \mathcal{I}^c_x(\phi) \end{array}$$

  where $\mathcal{I}^c_x$ is as $\mathcal{I}$, except that variable x is mapped into individual c
- Definitions of $\mathcal{I} \models \phi$, $\mathcal{I} \models \mathcal{T}$, $\models \phi$, $\mathcal{T} \models \phi$, $||\phi||_{\mathcal{T}}$ and $|\phi|_{\mathcal{T}}$ are as for the propositional case

Fuzzy RDF (we generalize [15, 16, 34])

- Statement (triples) may have attached a degree in $[0, 1]$: for $n \in [0, 1]$

$$\langle (subject, predicate, object), n \rangle$$

- Meaning: the degree of truth of the statement is at least n
- For instance,

$$\langle (o1, IsAbout, snoopy), 0.8 \rangle$$

Inferences in Fuzzy RDFS

Some inferences in fuzzy RDFS (set is not complete). Recall Rational Pavelka Logic ($\rightarrow$ is r-implication)

$$\frac{\langle (a, sp, b), n\rangle, \langle (b, sp, c), m\rangle}{(\langle (a, sp, c), n \wedge m\rangle} \qquad \frac{\langle (a, sp, b), n\rangle, \langle (x, a, y), m\rangle}{\langle (x, b, y), n \wedge m\rangle}$$

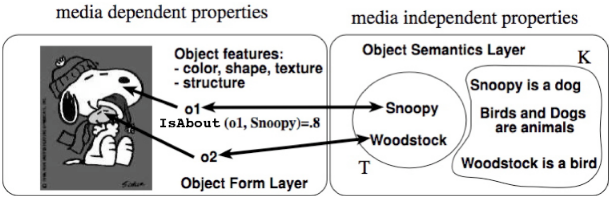
$$\frac{\langle (a, sc, b), n\rangle, \langle (b, sc, c), m\rangle}{\langle (a, sc, c), n \wedge m\rangle} \qquad \frac{\langle (a, sc, b), n\rangle, \langle (x, type, a), m\rangle}{\langle (x, type, b), n \wedge m\rangle}$$

$$\frac{\langle (a, dom, b), n\rangle, \langle (x, a, y), m\rangle}{\langle (x, type, b), n \wedge m\rangle} \qquad \frac{\langle (a, range, b), n\rangle, \langle (x, a, y), m\rangle}{\langle (y, type, b), n \wedge m\rangle}$$

$$\frac{\langle (a, dom, b), n\rangle, \langle (c, sp, a), m\rangle, \langle (x, c, y), k\rangle}{\langle (x, type, b), n \wedge m \wedge k\rangle} \qquad \frac{\langle (a, range, b), n\rangle, \langle (c, sp, a), m\rangle, \langle (x, c, y), k\rangle}{\langle (y, type, b), n \wedge m \wedge k\rangle}$$

sp = “subPropertyOf”, sc = “subClassOf”

Example



- Fuzzy RDF representation

$$\begin{array}{l}\langle (o1, IsAbout, snoopy), 0.8\rangle \\ \langle (snoopy, type, dog), 1.0\rangle \\ \langle (woodstock, type, bird), 1.0\rangle \\ \langle (dog, subClassOf, Animal), 1.0\rangle \\ \langle (bird, subClassOf, Animal), 1.0\rangle\end{array}$$

- then

$$KB \models \langle \exists x.(o1, IsAbout, x) \wedge (x, type, Animal), 0.8\rangle$$

Fuzzy DLs Basics [26]

The semantics is an immediate consequence of the First-Order-Logic translation of DLs expressions

Interpretation:

$$\mathcal{I} = \Delta^{\mathcal{I}}$$
$$C^{\mathcal{I}} : \Delta^{\mathcal{I}} \to [0,1]$$
$$R^{\mathcal{I}} : \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}} \to [0,1]$$

$\wedge$ = t-norm
$\vee$ = s-norm
$\neg$ = negation
$\rightarrow$ = implication

Concepts:

Syntax		Semantics		
$C, D \longrightarrow$	$\top \mid$	$\top^{\mathcal{I}}(x)$	$=$	1
	$\bot \mid$	$\bot^{\mathcal{I}}(x)$	$=$	0
	$A \mid$	$A^{\mathcal{I}}(x)$	$\in$	$[0,1]$
	$C \sqcap D \mid$	$(C_1 \sqcap C_2)^{\mathcal{I}}(x)$	$=$	$C_1{}^{\mathcal{I}}(x) \wedge C_2{}^{\mathcal{I}}(x)$
	$C \sqcup D \mid$	$(C_1 \sqcup C_2)^{\mathcal{I}}(x)$	$=$	$C_1{}^{\mathcal{I}}(x) \vee C_2{}^{\mathcal{I}}(x)$
	$\neg C \mid$	$(\neg C)^{\mathcal{I}}(x)$	$=$	$\neg C^{\mathcal{I}}(x)$
	$\exists R.C \mid$	$(\exists R.C)^{\mathcal{I}}(x)$	$=$	$\sup_{y \in \Delta^{\mathcal{I}}} R^{\mathcal{I}}(x,y) \wedge C^{\mathcal{I}}(y)$
	$\forall R.C$	$(\forall R.C)^{\mathcal{I}}(u)$	$=$	$\inf_{y \in \Delta^{\mathcal{I}}} R^{\mathcal{I}}(x,y) \rightarrow C^{\mathcal{I}}(y)\}$

Assertions: $\langle a{:}C, r\rangle$, $\mathcal{I} \models \langle a{:}C, r\rangle$ iff $C^{\mathcal{I}}(a^{\mathcal{I}}) \geq r$ (similarly for roles)

- individual a is instance of concept C at least to degree r, $r \in [0,1] \cap \mathbb{Q}$

Inclusion axioms: $C \sqsubseteq D$,

- $\mathcal{I} \models C \sqsubseteq D$ iff $\forall x \in \Delta^{\mathcal{I}}.C^{\mathcal{I}}(x) \leq D^{\mathcal{I}}(x)$
- this is equivalent to, $\forall x \in \Delta^{\mathcal{I}}.(C^{\mathcal{I}}(x) \rightarrow D^{\mathcal{I}}(x)) = 1$, if $\rightarrow$ is an r-implication

Basic Inference Problems

Consistency: Check if knowledge is meaningful

- Is KB consistent, i.e. satisfiable?

Subsumption: structure knowledge, compute taxonomy

- $KB \models C \sqsubseteq D$?

Equivalence: check if two fuzzy concepts are the same

- $KB \models C = D$?

Graded instantiation: Check if individual a instance of class C to degree at least r

- $KB \models \langle a{:}C, r\rangle$?

BTVB: Best Truth Value Bound problem

- $|a{:}C|_{KB} = \sup\{r \mid KB \models \langle a{:}C, r\rangle\}$?

Top-k retrieval: Retrieve the top-k individuals that instantiate C w.r.t. best truth value bound

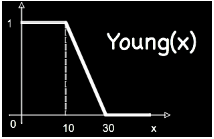
- $ans_{top-k}(KB, C) = Top_k\{\langle a, v\rangle \mid v = |a{:}C)|_{KB}\}$

Towards fuzzy OWL Lite and OWL DL

- Recall that OWL Lite and OWL DL relate to $\mathcal{SHIF}(D)$ and $\mathcal{SHOIN}(D)$, respectively
- We need to extend the semantics of fuzzy $\mathcal{ALC}$ to fuzzy $\mathcal{SHOIN}(D) = \mathcal{ALCHOINR}_+(D)$
- Additionally, we add
  - modifiers (e.g., *very*)
  - concrete fuzzy concepts (e.g., *Young*)
  - both additions have explicit membership functions

Concrete fuzzy concepts

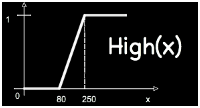
- E.g., *Small*, *Young*, *High*, *etc*. with explicit membership function
- Use the idea of concrete domains:
  - $D = \langle \Delta_D, \Phi_D \rangle$
  - Δ_D is an interpretation domain
  - Φ_D is the set of concrete fuzzy domain predicates d with a predefined arity $n = 1, 2$ and fixed interpretation $d^D \colon \Delta_D^n \to [0, 1]$
  - For instance,



$$\begin{array}{rcl} Minor & = & Person \sqcap \exists hasAge.\leq_{18} \\ YoungPerson & = & Person \sqcap \exists hasAge.Young \\ & & functional(hasAge) \end{array}$$

Modifiers

- *Very*, *moreOrLess*, *slightly*, etc.
- Apply to fuzzy sets to change their membership function
  - $very(x) = x^2$
  - $slightly(x) = \sqrt{x}$
- For instance,



$$SportsCar = Car \sqcap \exists speed.very(High)$$

Fuzzy $\mathcal{SHOIN}(D)$

Concepts:

	Syntax		Semantics
C, D	$\longrightarrow$	$\top$ |	$\top(x)$
		$\bot$ |	$\bot(x)$
		A |	$A(x)$
		$(C \sqcap D)$ |	$C_1(x) \wedge C_2(x)$
		$(C \sqcup D)$ |	$C_1(x) \vee C_2(x)$
		$(\neg C)$ |	$\neg C(x)$
		$(\exists R.C)$ |	$\exists x\ R(x, y) \wedge C(y)$
		$(\forall R.C)$ |	$\forall x\ R(x, y) \rightarrow C(y)$
		$\{a\}$ |	$x = a$
		$(\geq n\ R)$ |	$\exists y_1, \ldots, y_n. \bigwedge_{i=1}^{n} R(x, y_i) \wedge \bigwedge_{1 \leq i < j \leq n} y_i \neq y_j$
		$(\leq n\ R)$ |	$\forall y_1, \ldots, y_{n+1}. \bigwedge_{i=1}^{n+1} R(x, y_i) \rightarrow \bigvee_{1 \leq i < j \leq n+1} y_i = y_j$
		FCC |	$\mu_{FCC}(x)$
		$M(C)$ |	$\mu_M(C(x))$
R	$\longrightarrow$	P |	$P(x, y)$
		P^- |	$P(y, x)$

Assertions:

	Syntax		Semantics
α	$\longrightarrow$	$\langle a{:}C, r\rangle$ |	$C(a) \geq r$
		$\langle (a, b){:}R, r\rangle$	$R(a, b) \geq r$

Axioms:

	Syntax		Semantics
τ	$\longrightarrow$	$\langle C \sqsubseteq D, r\rangle$ |	$\forall x\ (C(x) \rightarrow D(x)) \geq r,$
		$fun(R)$ |	$\forall x \forall y \forall z\ R(x, y) \wedge R(x, z) \rightarrow y = z$
		$trans(R)$	$(\exists z\ R(x, z) \wedge R(z, y)) \rightarrow R(x, y)$

Example (Graded Entailment)

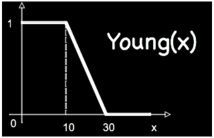
		
audi_tt	*mg*	*ferrari_enzo*

Car	*speed*
audi_tt	243
mg	≤ 170
ferrari_enzo	≥ 350

$$SportsCar \quad = \quad Car \sqcap \exists hasSpeed.very(High)$$

$$\begin{array}{lcl} KB & \models & \langle ferrari\_enzo{:}SportsCar, 1\rangle \\ KB & \models & \langle audi\_tt{:}SportsCar, 0.92\rangle \\ KB & \models & \langle mg{:}\neg SportsCar, 0.72\rangle \end{array}$$

Example (Graded Subsumption)

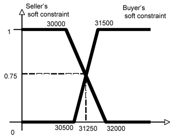


$$\begin{array}{rcl} Minor & = & Person \sqcap \exists hasAge. \leq_{18} \\ YoungPerson & = & Person \sqcap \exists hasAge.Young \end{array}$$

$$KB \models \langle Minor \sqsubseteq YoungPerson, 0.2 \rangle$$

Note: without an explicit membership function of *Young*, this inference cannot be drawn

Example (Simplified Negotiation)

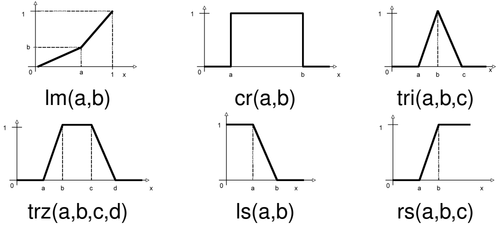


- a car seller sells an Audi TT for 31500 €, as from the catalog price.
- a buyer is looking for a sports-car, but wants to to pay not more than around 30000 €
- classical DLs: the problem relies on the crisp conditions on price
- more fine grained approach: to consider prices as fuzzy sets (as usual in negotiation)
  - seller may consider optimal to sell above 31500 €, but can go down to 30500 €
  - the buyer prefers to spend less than 30000 €, but can go up to 32000 €

$$AudiTT = SportsCar \sqcap \exists hasPrice.rs(30500, 31500)$$
$$Query = SportsCar \sqcap \exists hasPrice.ls(30000, 32000)$$

  - highest degree to which the concept
    $C = AudiTT \sqcap Query$
    is satisfiable is 0.75 (the possibility that the Audi TT and the query matches is 0.75)
  - the car may be sold at 31250 €

- Modifiers are definable as linear in-equations over $\mathbb{Q}, \mathbb{Z}$ (e.g., linear hedges), for instance, linear hedges, $lm(a, b)$, e.g. $very = lm(0.7, 0.49)$
- Fuzzy concrete concepts are definable as linear in-equations over $\mathbb{Q}, \mathbb{Z}$ (e.g., crisp, triangular, trapezoidal, left shoulder and right shoulder membership functions)



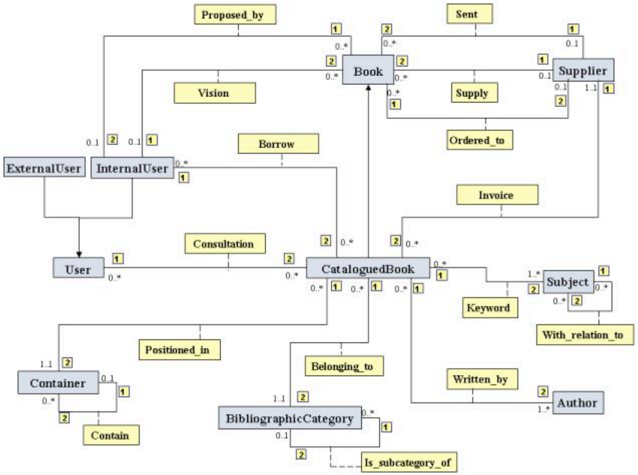
Implementation issues

- Several options exists:
  - Try to map fuzzy DLs to classical DLs
    - difficult to work with modifiers and concrete fuzzy concepts
  - Try to map fuzzy DLs to some fuzzy logic programming framework
    - A lot of work exists about mappings among classical DLs and LPs
    - But, needs a theorem prover for fuzzy LPs
  - Build an ad-hoc theorem prover for fuzzy DLs, using e.g., MILP
- A theorem prover for fuzzy $\mathcal{SHIF}$ + linear hedges + concrete fuzzy concepts + linear equational constraints + datatypes, under classical, Zadeh, Lukasiewicz and Product t-norm semantics has been implemented (`http://gaia.isti.cnr.it/~straccia`)
- FIRE: a fuzzy DL theorem prover for fuzzy $\mathcal{SHIN}$ under Zadeh semantics (`http://www.image.ece.ntua.gr/~nsimou/`)

Top-*k* retrieval in tractable DLs: the case of DL-Lite/DLR-Lite [25, 30]

- DL-Lite/DLR-Lite [3]: a simple, but interesting DLs
- Captures important subset of UML/ER diagrams
- Computationally tractable DL to query large databases
- Sub-linear, i.e. LOGSpace in data complexity
    - (same cost as for SQL)
- Good for very large database tables, with limited declarative schema design

- Knowledge base: $KB = \langle \mathcal{T}, \mathcal{A} \rangle$, where $\mathcal{T}$ and $\mathcal{A}$ are finite sets of axioms and assertions
- Axiom: $Cl \sqsubseteq Cr$ (inclusion axiom)
- Note for inclusion axioms: the language for left hand side is different from the one for right hand side
- DL-Lite$_{core}$:
  - Concepts: $$\begin{array}{rcl} Cl & \rightarrow & A \mid \exists R \\ Cr & \rightarrow & A \mid \exists R \mid \neg A \mid \neg \exists R \\ R & \rightarrow & P \mid P^{-} \end{array}$$
  - Assertion: $a{:}A$, $(a, b){:}P$
- DLR-Lite$_{core}$: (n-ary roles)
  - Concepts: $$\begin{array}{rcl} Cl & \rightarrow & A \mid \exists P[i] \\ Cr & \rightarrow & A \mid \exists P[i] \mid \neg A \mid \neg \exists P[i] \end{array}$$
  - $\exists P[i]$ is the projection on i-th column
  - Assertion: $a{:}A$, $\langle a_1, \ldots, a_n \rangle{:}P$
- Assertions are stored in relational tables
- Conjunctive query: $q(\mathbf{x}) \leftarrow \exists \mathbf{y}.conj(\mathbf{x}, \mathbf{y})$
  $conj$ is an aggregation of expressions of the form $B(z)$ or $P(z_1, z_2)$,


Proposed_by
Sent
Book
Supplier
Vision
Supply
Ordered_to
Borrow
ExternalUser
InternalUser
Invoice
User
Consultation
CataloguedBook
Subject
Keyword
With_relation_to
Positioned_in
Belonging_to
Written_by
Container
Author
BibliographicCategory
Contain
Is_subcategory_of

- Examples:

isa	$CatalogueBook \sqsubseteq Book$
disjointness	$Book \sqsubseteq \neg Author$
constraints	$CatalogueBook \sqsubseteq \exists positioned\_In$
role − typing	$\exists positioned\_In \sqsubseteq Container$
functional	$fun(positioned\_In)$
constraints	$Author \sqsubseteq \exists written\_By^{-}$
	$\exists written\_By \sqsubseteq CatalogueBook$
assertion	$Romeo\_and\_Juliet{:}CatalogueBook$
	$(Romeo\_and\_Juliet, Shakespeare){:}written\_By$
query	$q(x, y) \leftarrow CataloguedBook(x), Ordered\_to(x, y)$

- Consistency check is linear time in the size of the KB
- Query answering in linear in in the size of the number of assertions

Top-k retrieval in DL-Lite/DLR-Lite

- We extend the query formalism: conjunctive queries, where fuzzy predicates may appear
- conjunctive query

$$q(\mathbf{x}, s) \leftarrow \exists \mathbf{y}.conj(\mathbf{x}, \mathbf{y}), s = f(p_1(\mathbf{z}_1), \ldots, p_n(\mathbf{z}_n))$$

1. $\mathbf{x}$ are the *distinguished variables*;
2. s is the *score variable*, taking values in $[0, 1]$;
3. $\mathbf{y}$ are existentially quantified variables, called *non-distinguished variables*;
4. $conj(\mathbf{x}, \mathbf{y})$ is a conjunction of DL-Lite/DLR-Lite atoms $R(\mathbf{z})$ in *KB*;
5. $\mathbf{z}$ are tuples of constants in *KB* or variables in $\mathbf{x}$ or $\mathbf{y}$;
6. $\mathbf{z_i}$ are tuples of constants in *KB* or variables in $\mathbf{x}$ or $\mathbf{y}$;
7. p_i is an n_i-ary *fuzzy predicate* assigning to each n_i-ary tuple $\mathbf{c}_i$ the *score* $p_i(\mathbf{c}_i) \in [0, 1]$;
8. f is a monotone *scoring* function $f \colon [0, 1]^n \rightarrow [0, 1]$, which combines the scores of the n fuzzy predicates $p_i(\mathbf{c}_i)$

Example:

$$\begin{array}{rcl} Hotel & \sqsubseteq & \exists HasHLoc \\ Hotel & \sqsubseteq & \exists HasHPrice \\ Conference & \sqsubseteq & \exists HasCLoc \\ Hotel & \sqsubseteq & \neg Conference \end{array}$$

HasHLoc		*HasCLoc*		*HasHPrice*	
HotelID	*HasLoc*	*ConfID*	*HasLoc*	*HotelID*	*Price*
*h*1	*hl*1	*c*1	*cl*1	*h*1	150
*h*2	*hl*2	*c*2	*cl*2	*h*2	200
⋮	⋮	⋮	⋮	⋮	⋮

$$q(h, s) \leftarrow HasHLoc(h, hl), HasHPrice(h, p), Distance(hl, cl, d) \\ HasCLoc(c1, cl), s = cheap(p) \cdot close(d) .$$

where the fuzzy predicates *cheap* and *close* are defined as

$$\begin{array}{rcl} close(d) & = & ls(0, 2km, d) \\ cheap(p) & = & ls(0, 300, p) \end{array}$$

Tool exists and implemented in the DLMedia system

http://gaia.isti.cnr.it/~straccia

DLMedia: a Multimedia Information Retrieval System [33]

- Based on fuzzy DLR-Lite with similarity predicates
  - Axioms: $Rl_1 \sqcap \ldots \sqcap Rl_m \sqsubseteq Rr$

$$
\begin{array}{lll}
Rr & \longrightarrow & A \mid \exists[i_1, \ldots, i_k]R \\
Rl & \longrightarrow & A \mid \exists[i_1, \ldots, i_k]R \mid \exists[i_1, \ldots, i_k]R.(Cond_1 \sqcap \ldots \sqcap Cond_l) \\
Cond & \longrightarrow & ([i] \leq v) \mid ([i] < v) \mid ([i] \geq v) \mid ([i] > v) \mid ([i] = v) \mid ([i] \neq v) \mid \\
 & & ([i]\ simTxt\ 'k_1, \ldots, k_n') \mid ([i]\ simImg\ URN)
\end{array}
$$

  - $\exists[i_1, \ldots, i_k]R$ is the projection of the relation R on the columns $i_1, \ldots, i_k$
  - $\exists[i_1, \ldots, i_k]R.(Cond_1 \sqcap \ldots \sqcap Cond_l)$ further restricts the projection $\exists[i_1, \ldots, i_k]R$ according to the conditions specified in $Cond_i$
  - $([i]\ simTxt\ 'k_1 \ldots k_n')$ evaluates the degree of being the text of the i-th column similar to the list of keywords $k_1 \ldots k_n$
  - $([i]\ simImg\ URN)$ returns the system's degree of being the image identified by the i-th column similar to the image identified by the URN
  - Facts: $\langle R(c_1, \ldots, c_n), s\rangle$

- Example axioms

$\exists[1,2]Person \sqsubseteq \exists[1,2]hasAge$
// constrains relation $hasAge(name, age)$
$\exists[3,1]Person \sqsubseteq \exists[1,2]hasChild$
// constrains relation $hasChild(father\_name, name)$
$\exists[4,1]Person \sqsubseteq \exists[1,2]hasChild$
// constrains relation $hasChild(mother\_name, name)$
$\exists[3,1]Person.(([2] \geq 18) \sqcap ([5] =' female')) \sqsubseteq \exists[1,2]hasAdultDaughter$
// constrains relation $hasAdultDaughter(father\_name, name)$

- On the other hand examples axioms involving similarity predicates are,

$$\exists[1]ImageDescr.([2]\ simImg\ urn1) \sqsubseteq Child \quad (1)$$
$$\exists[1]Title.([2]\ simTxt\ 'lion') \sqsubseteq Lion \quad (2)$$

where $urn1$ identifies the image

- Example queries

$q(x) \leftarrow Child(x)$
// find objects about a child (strictly speaking, find instances of *Child*)

$q(x) \leftarrow CreatorName(x, y) \land (y =' paolo'), Title(x, z), (z\ simTxt\ 'tour')$
// find images made by Paolo whose title is about 'tour'

$q(x) \leftarrow ImageDescr(x, y) \land (y\ simImg\ urn2)$
// find images similar to a given image identified by *urn*2

$q(x) \leftarrow ImageObject(x) \land isAbout(x, y_1) \land Car(y_1) \land isAbout(x, y_2) \land Racing(y_2)$
// find image objects about cars racing

Fuzzy LPs Basics [4, 6, 7, 22, 23, 29, 35]

- Many Logic Programming (LP) frameworks have been proposed to manage uncertain and imprecise information. They differ in:
  - The underlying notion of uncertainty and vagueness: probability, possibility, many-valued, fuzzy logics
  - How values, associated to rules and facts, are managed
- We consider fuzzy LPs, where
  - Truth space is $[0, 1]$
  - Interpretation is a mapping $I : B_{\mathcal{P}} \to [0, 1]$
  - Generalized LP rules are of the form

$$R(\mathbf{x}) \leftarrow \exists \mathbf{y}.f(R_1(\mathbf{z}_1), \ldots, R_l(\mathbf{z}_l), p_1(\mathbf{z}'_1), \ldots, p_h(\mathbf{z}'_h)) \ ,$$

    - Meaning of rules: "take the truth-values of all $R_i(\mathbf{z}_i), p_j(\mathbf{z}'_j)$, combine them using the truth combination function f, and assign the result to $R(\mathbf{x})$"

- Same meaning as for fuzzy DLR-Lite queries

$$R(\mathbf{x}, s) \leftarrow \exists \mathbf{y}.conj(\mathbf{x}, \mathbf{y}), s = f(p_1(\mathbf{z}_1), \ldots, p_{l+h}(\mathbf{z}_{l+h}))$$

1. $\mathbf{x}$ are the *distinguished variables*;
2. s is the *score variable*, taking values in $[0, 1]$;
3. $\mathbf{y}$ are existentially quantified variables, called *non-distinguished variables*;
4. $conj(\mathbf{x}, \mathbf{y})$ is a list of atoms $R_i(\mathbf{z})$ in *KB*;
5. $\mathbf{z}$ are tuples of constants in *KB* or variables in $\mathbf{x}$ or $\mathbf{y}$;
6. $\mathbf{z_i}$ are tuples of constants in *KB* or variables in $\mathbf{x}$ or $\mathbf{y}$;
7. p_i is an n_i-ary *fuzzy predicate* assigning to each n_i-ary tuple $\mathbf{c}_i$ the *score* $p_i(\mathbf{c}_i) \in [0, 1]$;
8. f is a monotone *scoring* function $f\colon [0, 1]^{l+h} \to [0, 1]$, which combines the scores of the n fuzzy predicates $p_i(\mathbf{c}_i)$

Example: Soft shopping agent

- I may represent my preferences in Logic Programming with the rules

$$
\begin{array}{rcl}
Pref_1(x,p,s) & \leftarrow & HasPrice(x,p), LS(10000, 14000, p, s) \\
Pref_2(x,s) & \leftarrow & HasKM(x,k), LS(13000, 17000, k, s) \\
Buy(x,p,s) & \leftarrow & Pref_1(x,p,s_1), Pref_2(x,s_2), s = 0.7 \cdot s_1 + 0.3 \cdot s_2
\end{array}
$$

ID	MODEL	PRICE	KM
455	MAZDA 3	12500	10000
34	ALFA 156	12000	15000
1812	FORD FOCUS	11000	16000
⋮	⋮	⋮	⋮

- Problem: All tuples of the database have a score:
  - We cannot compute the score of all tuples, then rank them. Brute force approach not feasible.
- Top-k problem: Determine efficiently just the top-k ranked tuples, without evaluating the score of all tuples. E.g. top-3 tuples

ID	PRICE	SCORE
1812	11000	0.6
455	12500	0.56
34	12000	0.50

Top-*k* retrieval in LPs

- If the database contains a huge amount of facts, a brute force approach fails:
  - one cannot anymore compute the score of all tuples, rank all of them and only then return the top-*k*
- Better solutions exists for restricted fuzzy LP languages: Datalog + restriction on the score combination functions appearing in the body [29, 32]

Fuzzy DLPs Basics [10, 11, 27, 31]

- Combine fuzzy DLs with fuzzy LPs:
  - Like fuzzy LPs, but DL atoms and roles may appear in rules

$$\begin{array}{lcl} \mathit{LowCarPrice}(z) & \leftarrow & \min(\mathit{made\_by}(x,y), \mathit{DL}[\mathit{ChineseCarCompany}](y) \\ & & \mathit{price}(x,z)) \cdot \mathit{DL}[\mathit{Low}](z) \\ \\ \mathit{Low} & = & \mathit{LS}(5.000, 15.000) \\ \mathit{ChineseCarCompany} & \sqsubseteq & \exists \mathit{has\_location}.\mathit{China} \end{array}$$

- Knowledge Base is a pair $KB = \langle \mathcal{P}, \Sigma \rangle$, where
  - $\mathcal{P}$ is a fuzzy logic program
  - Σ is a fuzzy DL knowledge base (set of assertions and inclusion axioms)

Fuzzy DLPs Semantics

- Semantics: several approaches
- In principle, for each classical semantics based integration between DLs and LPs, there is be a fuzzy analogue
  - Pay attention, the fuzzy variant may add further technical and computational complications

1. Axiomatic approach: fuzzy DL atoms and roles are managed uniformely
2. Loosely Coupled approach: fuzzy DL atoms and roles are like “procedural attachments” (procedural calls to a fuzzy DL theorem prover)
3. Tightly coupled approach: The DL component restricts the models to be considered for the LP component

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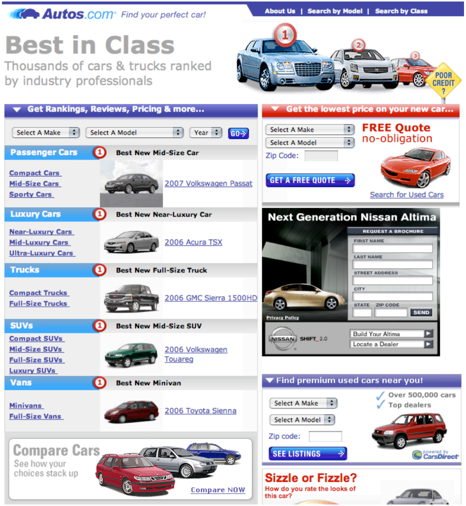
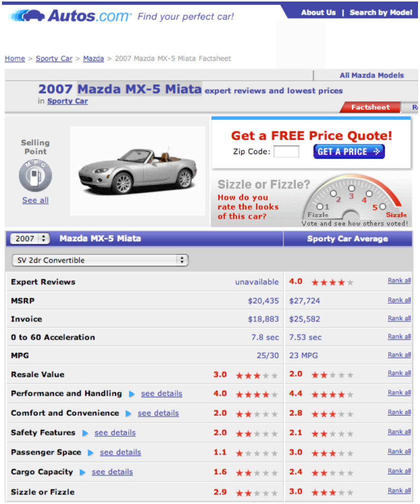
- Description logic programs that allow for dealing with probabilistic uncertainty and fuzzy vagueness.
- Semantically, probabilistic uncertainty can be used for data integration and ontology mapping, and fuzzy vagueness can be used for expressing vague concepts.
- Technically, allows for defining different rankings on ground atoms using fuzzy vagueness, and then for a probabilistic merging of these rankings using probabilistic uncertainty.
- Query processing based on fixpoint iterations.

Suppose a person would like to buy “a sports car that costs at most about 22 000 euro and that has a power of around 150 HP”.

In todays Web, the buyer has to *manually*

- search for car selling web sites, e.g., using Google;
- select the most promising sites;
- browse through them, query them to see the cars that each site sells, and match the cars with the requirements;
- select the offers in each web site that match the requirements; and
- eventually merge all the best offers from each site and select the best ones.


Autos.com Find your perfect car!
About Us | Search by Model | Search by Class
Best in Class
Thousands of cars & trucks ranked by industry professionals
POOR CREDIT ?
Get Rankings, Reviews, Pricing & more...
Select A Make
Select A Model
Year
GO
Passenger Cars
Best New Mid-Size Car
Compact Cars
Mid-Size Cars
Sporty Cars
2007 Volkswagen Passat
Luxury Cars
Best New Near-Luxury Car
Near-Luxury Cars
Mid-Luxury Cars
Ultra-Luxury Cars
2006 Acura TSX
Trucks
Best New Full-Size Truck
Compact Trucks
Full-Size Trucks
2006 GMC Sierra 1500HD
SUVs
Best New Mid-Size SUV
Compact SUVs
Mid-Size SUVs
Full-Size SUVs
Luxury SUVs
2006 Volkswagen Touareg
Vans
Best New Minivan
Minivans
Full-Size Vans
2006 Toyota Sienna
Compare Cars
See how your choices stack up
Compare NOW
Get the lowest price on your new car...
Select A Make
Select A Model
Zip Code:
FREE Quote
no-obligation
GET A FREE QUOTE
Search for Used Cars
Next Generation Nissan Altima
REQUEST A BROCHURE
FIRST NAME
LAST NAME
STREET ADDRESS
CITY
STATE ZIP CODE
SEND
Privacy Policy
NISSAN SHIFT_2.0
Build Your Altima
Locate a Dealer
Find premium used cars near you!
Over 500,000 cars
Top dealers
Select A Make
Select A Model
Zip code:
SEE LISTINGS
powered by CarsDirect
Sizzle or Fizzle?
How do you rate the looks of this car?



2007	Mazda MX-5 Miata		Sporty Car Average		
Expert Reviews		unavailable	4.0	★★★★	Rank all
MSRP		$20,435	$27,724		Rank all
Invoice		$18,883	$25,582		Rank all
0 to 60 Acceleration		7.8 sec	7.53 sec		Rank all
MPG		25/30	23 MPG		Rank all
Resale Value	3.0	★★★	2.0	★★	Rank all
Performance and Handling ▶ see details	4.0	★★★★	4.4	★★★★	Rank all
Comfort and Convenience ▶ see details	2.0	★★	2.8	★★★	Rank all
Safety Features ▶ see details	2.0	★★	2.1	★★	Rank all
Passenger Space ▶ see details	1.1	★	3.0	★★★	Rank all
Cargo Capacity ▶ see details	1.6	★★	2.4	★★	Rank all
Sizzle or Fizzle	2.9	★★	3.0	★★★	Rank all

A *shopping agent* may support us, *automatizing* the whole process once it receives the request/query q from the buyer:

- The agent selects some sites/resources S that it considers as *relevant* to q (represented by probabilistic rules).
- For the top-k selected sites, the agent has to reformulate q using the terminology/ontology of the specific car selling site (which is done using probabilistic rules).
- The query q may contain many *vague/fuzzy* concepts such as "the price is around 22 000 euro or less", and so a car may *match* q to a *degree*. So, a resource returns a ranked list of cars, where the ranks depend on the degrees to which the cars match q.
- Eventually, the agent integrates the ranked lists (using probabilities) and shows the top-n items to the buyer.

$$
\begin{array}{l}
\mathit{Cars} \sqcup \mathit{Trucks} \sqcup \mathit{Vans} \sqcup \mathit{SUVs} \sqsubseteq \mathit{Vehicles} \\
\mathit{PassengerCars} \sqcup \mathit{LuxuryCars} \sqsubseteq \mathit{Cars} \\
\mathit{CompactCars} \sqcup \mathit{MidSizeCars} \sqcup \mathit{SportyCars} \sqsubseteq \mathit{PassengerCars}
\end{array}
$$

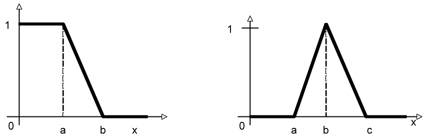
$$
\begin{array}{ll}
\mathit{Cars} \sqsubseteq & (\exists \mathit{hasReview}.\mathit{Integer}) \sqcap (\exists \mathit{hasInvoice}.\mathit{Integer}) \\
& \sqcap (\exists \mathit{hasResellValue}.\mathit{Integer}) \sqcap (\exists \mathit{hasMaxSpeed}.\mathit{Integer}) \\
& \sqcap (\exists \mathit{hasHorsePower}.\mathit{Integer}) \sqcap \ldots
\end{array}
$$

$$
\begin{array}{l}
\mathit{MazdaMX5Miata} \colon \mathit{SportyCar} \sqcap (\exists \mathit{hasInvoice}.18883) \\
\qquad \sqcap (\exists \mathit{hasHorsePower}.166) \sqcap \ldots \\
\mathit{MitsubishiEclipseSpyder} \colon \mathit{SportyCar} \sqcap (\exists \mathit{hasInvoice}.24029) \\
\qquad \sqcap (\exists \mathit{hasHorsePower}.162) \sqcap \ldots
\end{array}
$$

We may now encode "costs at most about 22 000 euro" and "has a power of around 150 HP" in the buyer's request through the following concepts C and D, respectively:

$$C = \exists hasInvoice.LeqAbout22000 \text{ and}$$
$$D = \exists hasHorsePower.Around150HP,$$

where $LeqAbout22000 = ls(22000, 25000)$ and $Around150HP = tri(125, 150, 175)$.



The following fuzzy dl-rule encodes the buyer's request "a sports car that costs at most about 22 000 euro and that has a power of around 150 HP".

$$
\begin{aligned}
query(x) \quad \leftarrow_{\otimes} \quad & SportyCar(x) \wedge_{\otimes} \\
& hasInvoice(x, y_1) \wedge_{\otimes} \\
& DL[LeqAbout22000](y_1) \wedge_{\otimes} \\
& hasHorsePower(x, y_2) \wedge_{\otimes} \\
& DL[Around150HP](y_2) \geq 1 \,.
\end{aligned}
$$

Here, $\otimes$ is the Gödel t-norm (that is, $x \otimes y = \min(x, y)$).

The buyer's request, but in a "different" terminology:

$$\begin{aligned} query(x) \leftarrow_\otimes\ & SportsCar(x) \wedge_\otimes hasPrice(x, y_1) \wedge_\otimes hasPower(x, y_2) \wedge_\otimes \\ & DL[LeqAbout22000](y_1) \wedge_\otimes DL[Around150HP](y_2) \geq 1 \end{aligned}$$

Ontology alignment mapping rules:

$$\begin{aligned} SportsCar(x) &\leftarrow_\otimes DL[SportyCar](x) \wedge_\otimes sc_{pos} \geq 0.9 \\ hasPrice(x) &\leftarrow_\otimes DL[hasInvoice](x) \wedge_\otimes hi_{pos} \geq 0.8 \\ hasPower(x) &\leftarrow_\otimes DL[hasHorsePower](x) \wedge_\otimes hhp_{pos} \geq 0.8\,, \end{aligned}$$

Probability distribution μ:

$$\begin{array}{ll} \mu(sc_{pos}) = 0.91 & \mu(sc_{neg}) = 0.09 \\ \mu(hi_{pos}) = 0.78 & \mu(hi_{neg}) = 0.22 \\ \mu(hhp_{pos}) = 0.83 & \mu(hhp_{neg}) = 0.17\,. \end{array}$$

The following are some tight consequences:

$$KB \;\Vert\!\!\sim_{\mathit{tight}}\; (\mathbf{E}[\mathit{query}((\mathit{MazdaMX5Miata})])[0.21, 0.21]$$
$$KB \;\Vert\!\!\sim_{\mathit{tight}}\; (\mathbf{E}[\mathit{query}((\mathit{MitsubishiEclipseSpyder})])[0.19, 0.19]\,.$$

Informally, the expected degree to which *MazdaMX5Miata* matches the query q is 0.21, while the expected degree to which *MitsubishiEclipseSpyder* matches the query q is 0.19,

Thus, the shopping agent ranks the retrieved items as follows:

rank	item	degree
1.	*MazdaMX5Miata*	0.21
2.	*MitsubishiEclipseSpyder*	0.19