Notes on Markovian Extension of a Dialect of Value Passing CCS [⋆]

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1 Introduction and motivation

Many calculi for Service Oriented Computing (SOC) are based on a two-party, CCS-like, communication paradigm (see e.g. [1, 2, 9]). In the context of SOC architectures, *code mobility* and *dynamic* process creation will play a major role, especially in distributed environments. In this framework, properties like commutativity and associativity of parallel composition operators is highly desirable.

In this note we show how a relevant subset of CCS with value passing can be extended with stochastic information in such a way that associativity and commutativity of process parallel compositions is preserved, in the sense that $P|(Q|R) \sim_M (P|Q)|R$ and $P|Q \sim_M Q|P$, where $\sim_M$ denotes Strong Markovian Bisimulation Equivalence.

2 Rationale

One of the major issues in the definition of action synchronisation composition operators of stochastic calculi is what value to choose for the rate of the synchronisation, as a function of the rates of the synchronising actions. The approach proposed by Hillston [8], originally developed for PEPA multi-party, CSP-like synchronisation, is by now widely accepted. The key choices there are (i) that the rate of the *slowest* component is chosen as the rate of the synchronisation, and (ii) the introduction of the notion of *apparent rates*. We briefly recall the approach below.

Let us consider the two processes P, defined as (α, r_p), and Q, defined as (α, r_q), i.e. both P and Q can perform only one action, α (and then stop), although, with different rates, namely r_p, and r_q respectively. According to the semantics of PEPA, their parallel composition, with synchronisation on α, i.e.:

$$P \bowtie_{\{\alpha\}} Q$$

will perform an α and the associated rate will be $min(r_p, r_q)$.

Let us now consider the case in which component P can perform α in several different ways, each with a possibly different rate, and similarly for Q. Considering P as a black-box, given the *race condition* principle, the observed rate for α is the sum of the rates of *all* the transitions labelled by α, α-transitions in the sequel, of P^1. The result is called the *apparent rate* of α in P, denoted by $r_\alpha(P)$, which can be computed recursively on the syntactical structure of PEPA terms and shown to satisfy the following equation:

$$r_\alpha(P) = \sum_{P \xrightarrow{(\alpha, r_j)} P_j \in Ts(P)} r_j$$

where $Ts(P)$ is the set of transitions enabled in P. Similarly, we define $r_\alpha(Q)$. The (apparent) rate of α in $P \bowtie_\alpha Q$ is thus given by:

$$min(r_\alpha(P), r_\alpha(Q)).$$

The problem remains of how to compute the rate of each individual α-transition in $P \bowtie_{\{\alpha\}} Q$ as a function of the rates of the specific α-transitions selected in the components P and Q which gave rise to the synchronisation. This is done by using the conditional probability of taking a *specific* α-transition in P, with rate, say, r_0, given that an α-transition is taken, which of course is equal to:

$$\frac{r_0}{r_\alpha(P)}.$$

Similarly, for Q, taking an α-transition with rate, say, r_1, we get a conditional probability

$$\frac{r_1}{r_\alpha(Q)}.$$

Finally, the rate of the α-transition in $P \bowtie_{\{\alpha\}} Q$ resulting from the synchronisation of the above transitions is:

$$\frac{r_0}{r_\alpha(P)} \cdot \frac{r_1}{r_\alpha(Q)} \cdot min(r_\alpha(P), r_\alpha(Q)).$$

Notice that, obviously, taking the sum over the rates of *all* α-transitions in $P \bowtie_{\{\alpha\}} Q$ we get $r_\alpha(P \bowtie_{\{\alpha\}} Q) = min(r_\alpha(P), r_\alpha(Q))$.

In the sequel, we adapt the approach sketched above to a simple value passing, CCS-like calculus, keeping in mind the major requirements arising from the Service Oriented framework, like preservation of associativity of the parallel composition operator.

Let us take a process P which can perform the output of a value v at channel a, with rate r_p, and then stop. We denote such a process by $(a!v, r_p).nil$. Similarly, let Q be a process which can perform an input from the same channel, with

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[1] In this section, we do not consider transition identity/multiplicity for the sake of simplicity. The examples are chosen in such a way that no ambiguities can arise.

rate r_q, i.e. $Q = (a?x, r_q).nil$. As in the previous case, the rate of the τ-transition, resulting from the interaction between P and Q in the parallel composition $P|Q$, is $min(r_p, r_q)$. Let us now consider a more general case, in which P can perform the output of value v at channel a in several different ways, and similarly for Q with the input from a. Assume, moreover, that P has *no* transition enabled by means of which it can perform an *input* over channel a; similarly, assume that Q has *no* transition enabled by means of which it can perform any *output* over a. In other words, P and Q can be written in the following form:

$$P = \sum_{i=1}^{n}(a_i!v_i, r_{p_i}).nil \qquad\qquad Q = \sum_{j=1}^{k}(a_j?x_j, r_{q_j}).nil$$

where $v_i = v$ and $a_i = a$ for some i, and, similarly, $a_j = a$ for some j. Also in this case, we can just take the same definition as in [8], using apparent rates and conditional probabilities for computing the rates of each individual τ-transition of $P|Q$ corresponding to an interaction on channel a involving value v. For example, if $P = (a!v, r_{p1}).nil + (a!v, r_{p2}.nil)$ and $Q = (a?x, r_{q1}).nil + (a?y, r_{q2}).nil$ the τ-transitions of $P|Q$ are:

$$P|Q \xrightarrow{\tau, \frac{r_{p1}}{r_{p1}+r_{p2}} \cdot \frac{r_{q1}}{r_{q1}+r_{q2}} \cdot min(r_{p1}+r_{p2}, r_{q1}+r_{q2})} nil|nil$$

$$P|Q \xrightarrow{\tau, \frac{r_{p1}}{r_{p1}+r_{p2}} \cdot \frac{r_{q2}}{r_{q1}+r_{q2}} \cdot min(r_{p1}+r_{p2}, r_{q1}+r_{q2})} nil|nil$$

$$P|Q \xrightarrow{\tau, \frac{r_{p2}}{r_{p1}+r_{p2}} \cdot \frac{r_{q1}}{r_{q1}+r_{q2}} \cdot min(r_{p1}+r_{p2}, r_{q1}+r_{q2})} nil|nil$$

$$P|Q \xrightarrow{\tau, \frac{r_{p2}}{r_{p1}+r_{p2}} \cdot \frac{r_{q2}}{r_{q1}+r_{q2}} \cdot min(r_{p1}+r_{p2}, r_{q1}+r_{q2})} nil|nil$$

Clearly, the sum of the rates of the above τ-transitions is again $min(r_{p1} + r_{p2}, r_{q1} + r_{q2})$, which is the minimum of the *total rate of output* of v over a, i.e. the total rate of $a!v$-transitions, and the *total rate of input* of v over a, i.e. the total rate of $a?v$-transitions.

The case now remains where at least one component is a parallel composition in turn where both input (from a) and output (of v over a) transitions are enabled. For example, $P = P_1|P_2$ where $P_1 = (a!v, r_{p1}).nil$ and $P_2 = (a?x, r_{p2}).nil$, while $Q = (a?y, r_q).nil$.

By just applying Hillston's approach, which works fine with CSP-like synchronisation, considering $a!v$ and $a?v$ as distinct actions which synchronise generating a τ-transition[2], the resulting parallel composition $(P_1|P_2)|Q$ has the following two τ-transitions and associated rates:

$$(P_1|P_2)|Q \xrightarrow{\tau(a,v), min(r_{p1}, r_{p2})} (nil|nil)|Q$$
$$(P_1|P_2)|Q \xrightarrow{\tau(a,v), min(r_{p1}, r_q)} (nil|P_2)|nil$$

where we use the notation $\tau(a, v)$ for recording that the specific synchronisation has taken place by means of sending/receiving value v over channel a. Let us

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[2] An early semantics is implicitly assumed here.

consider the first of the above two transitions. The associated rate $min(r_{p1}, r_{p2})$ is the result of

$$\frac{r_{p1}}{r_{a!v}(P_1)} \cdot \frac{r_{p2}}{r_{a?v}(P_2)} \cdot min(r_{p1}, r_{p2}).$$

where we let $r_{a!v}(R)$ and $r_{a?v}(R)$ denote the apparent rates of $a!v$ and $a?v$ in generic process R, and we get $r_{a!v}(P_1) = r_{p1}$, $r_{a?v}(P_2) = r_{p2}$, and $r_{a?v}(Q) = r_q$. The probability of selecting the specific transition of P_1 is $\frac{r_{p1}}{r_{a!v}(P_1)}$, whereas $\frac{r_{p2}}{r_{a?v}(P_2)}$ is the probability of selecting the specific transition of P_2. Notice that the probability of choosing the output action $a!v$ of process P_1 in the *complete* system at hand, namely $(P_1|P_2)|Q$, is of course $1 = \frac{r_{p1}}{r_{a!v}(P_1)}$. On the other hand, the probability of selecting the $a?v$-transition of P_2 for the interaction, in the complete system, is *not* $1 = \frac{r_{p2}}{r_{a?v}(P_2)}$, but rather $\frac{r_{p2}}{r_{a?v}(P_2)+r_{a?v}(Q)}$. In fact, there is a well known *key difference* between the synchronisation paradigm of CCS and that of CSP. In CSP, the $a!v$-transition of P_1 can synchronise *either* with the $a?v$-transition of P_2 *or* with that of Q, while, in the CSP framework the same transition would synchronise with *both* (thus realising a multicast from P_1 to P_2 and Q). This difference, has an impact on the probabilities as well.

In the general case of the parallel composition $R|S$ of two generic processes R and S, the probability that a generic $a!v$-transition in R, with rate r, is selected for synchronisation with a $a?v$-transition in S is obtained by dividing r by the *cumulative rate* of $a!v$ in the *parallel composition* $R|S$, i.e. the *sum* of the rates of *all* $a!v$-transitions $R|S$, which we denote by $c_{a!v}(R|S)$; in other words, the probability of interest is $\frac{r}{c_{a!v}(R|S)}$. A similar reasoning applies to the computation of the probability that a specific $a?v$-transition in S, with rate s, is selected for synchronisation in the parallel composition $R|S$, which yields $\frac{s}{c_{a?v}(R|S)}$. Notice that the above reasoning is correct only under the assumption that there is no choice operator with both input and output alternatives. Otherwise, one would consider in the cumulative rate also the rates of those actions which cannot interact because they are different alternatives of the same choice. For example, consider the process $((a!v, r_1).nil + (a?x, r_2).nil)|(a!v, r_3).nil$. In this expression, the cumulative rate of $a!v$ would amount to $r_1 + r_3$, while it is clear that the action with rate r_1 cannot participate in a synchronisation within the system. We therefore restrict the use of the choice operator to the following forms in the language:

$$P ::= \sum_{i=1}^{n}(a_i!v_i, r_{p_i}).P_i \mid \sum_{i=1}^{n}(a_i?x_i, r_{p_i}).P_i$$

Let us now take a closer look at the relevant rates. The rate at which a $a!v$-transition is executed starting from state $R|S$ is the cumulative rate $c_{a!v}(R|S)$. Similarly, the rate at which a $a?v$-transition is executed starting from state $R|S$ is $c_{a?v}(R|S)$. In a sense, cumulative rates in the CCS framework play a similar role as apparent rates in the CSP paradigm. The major difference is that cumulative rates refer to either input or output transitions, while the "polarity" of the transition is irrelevant in the case of apparent rates (in fact, there is no polarity notion in CSP!). Consequently, we would like the rate at which a

$\tau(a, v)$-transition is executed starting from state $R|S$ be

$$min(c_{a!v}(R|S), c_{a?v}(R|S))$$

Furthermore, if we put $R|S$ in a broader parallel composition context, we expect that the cumulative rates may increase.

This is exactly what we achieve in the operational semantics we define in the next section, where we follow an approach which is different from that of [10] for the stochastic π-Calculus. Technically, in the rule for process synchronisation $R|S$, we add four more parameters to the label of each $\tau(a, v)$-transition, which now takes the following form:

$$\tau(a, v, \lambda_O, \lambda_I, \mu_{cO}, \mu_{cI})$$

where λ_O is the rate of the specific $a!v$-transition involved in the interaction, λ_I is that of the related $a?v$-transition, μ_{cO} is the cumulative rate of $a!v$ in $R|S$, and μ_{cI} is the cumulative rate of $a?v$ in $R|S$. Cumulative rate manipulation (i.e. increase) is instead dealt with in the rule for process interleaving. Finally, the actual rate label for each transition is computed as the related entry of the rate-matrix $\mathcal{R}$ of the relevant CTMC:

$$\mathcal{R}[s, s'] \stackrel{def}{=} \sum_{s \xrightarrow{\tau(a,v,\lambda_O,\lambda_I,\mu_{cO},\mu_{cI})}_\gamma s'} \frac{\lambda_O}{\mu_{cO}} \cdot \frac{\lambda_I}{\mu_{cI}} \cdot min(\mu_{cO}, \mu_{cI})$$

In our simple example we get the following τ-transitions:

$(P_1|P_2)|Q \xrightarrow{\tau(a,v,r_{p1},r_{p2},r_{p1},r_{p2}+r_q)} (nil|nil)|Q$
$(P_1|P_2)|Q \xrightarrow{\tau(a,v,r_{p1},q,r_{p1},r_{p2}+r_q)} (nil|P_2)|nil$

The CTMC rate for the first transition is $\frac{r_{p1}}{r_{p1}} \cdot \frac{r_{p2}}{r_{p2}+r_q} \cdot min(r_{p1}, r_{p2}+r_q)$ while that for the second is $\frac{r_{p1}}{r_{p1}} \cdot \frac{r_q}{r_{p2}+r_q} \cdot min(r_{p1}, r_{p2} + r_q)$. The total exit rate of a τ-transition is $min(r_{p1}, r_{p2} + r_q)$, as expected.

We close this section by noting that, by using our approach, for generic (R_1, R_2), and R_3, the CTMC of $(R_1|R_2)|R_3$ is strong Markovian bisimulation equivalent to that of $R_1|(R_2|R_3)$, which is a highly desirable property in the context of mobile, dynamic processes like those of distributed SOC. Unfortunately, this associativity property would not hold when using directly the apparent rates approach in the CCS synchronisation paradigm, as in [10]. For instance, in our simple example, for $(P_1|P_2)|Q$, as we have seen, we would get the following transitions:

$(P_1|P_2)|Q \xrightarrow{\tau(a,v),min(r_{p1},r_{p2})} (nil|nil)|Q$
$(P_1|P_2)|Q \xrightarrow{\tau(a,v),min(r_{p1},r_q)} (nil|P_2)|nil$

while for $P_1|(P_2|Q)$ we would get the following transitions:

$$P_1|(P_2|Q) \xrightarrow{\tau(a,v),\frac{r_{p2}}{r_{p2}+r_q}min(r_{p1},r_{p2}+r_q)} nil|(nil|Q)$$

$$P_1|(P_2|Q) \xrightarrow{\tau(a,v),\frac{r_q}{r_{p2}+r_q}min(r_{p1},r_{p2}+r_q)} nil|(P_2|nil)$$

The CTMC of $(P_1|P_2)|Q$ and $P_1|(P_2|Q)$ are *not* Markovian bisimulation equivalent. This is in fact due to the different rates with which $(P_1|P_2)|Q$ and $P_1|(P_2|Q)$ go, for instance to Q, namely $min(r_{p1},r_{p2})$ and $\frac{r_{p2}}{r_{p2}+r_q}min(r_{p1},r_{p2}+r_q)$, respectively, which are in general not equal.

3 Formal definition

In this section we present the formal definition of the stochastic extension of the simple dialect of value passing CCS.

3.1 Syntax

Let V, ranged over by $v, v_i, v' \ldots$ be a set of *values*, Var, ranged over by $x, x_i, y \ldots$ be a set of *variables*, and $Chan$, ranged over by $a, a_i, b \ldots$ be a set of *channels*. We assume the above sets mutually disjoint and we let $\lambda, \lambda_i, \mu \ldots \in \mathbb{R}^+$ denote *rates* of exponentially distributed random variables. The syntax follows:

$$P, Q ::= \sum_{i=1}^k (a_i?x_i, \lambda_i).P_i \mid \sum_{i=1}^k (a_i!e_i, \lambda_i).P_i \mid P|P$$

$$e ::= v \mid x$$

The only variable binding operator is the input operator. All free occurrences of x_i in P_i are bound by $?x_i$ in $\sum_{i=1}^k (a_i?x_i, \lambda_i).P_i$. A term is *closed* if it contains no free occurrence of any variable. We let $\mathcal{C}$ denote the set of all closed terms generated by the above grammar.

3.2 Semantics

In the following we define the operational semantics for closed terms P. They associate a CTMC to each closed term P; this is done via the definition of a LTS from which the CTMC is then generated. The set of possible labels is defined by the following grammar:

$$\Gamma ::= (\alpha, \lambda) \mid \tau(a, v, \lambda_O, \lambda_I, \mu_{cO}, \mu_{cI})$$

where:

$$\alpha ::= a?v \mid a!v$$

A label of the form $a?v$ ($a!v$, resp.) denotes the offer to receive (send, resp.) value v over channel a. A label of the form $\tau(a, v, \lambda_O, \lambda_I, \mu_{cO}, \mu_{cI})$ represents instead an *interaction*, where value v is exchanged via channel a; the rate associated to the send action $a!v$ involved in the interaction is λ_O, while the one associated to the receive action $a?v$ is λ_I. Similarly, the *cumulative* rate for $a!v$ is μ_{cO} and that for $a?v$ is μ_{cI}.

For each α and P, the *cumulative rate* function c_α is defined recursively on the structure of the syntax of P, as follows:

$$c_{a?v}(\textstyle\sum_{i=1}^{k}(a_i?x_i, \lambda_i).P_i) \stackrel{def}{=} \sum_{i \in \{j | 1 \leq j \leq k, a_j = a\}} \lambda_i$$

$$c_{a?v}(\textstyle\sum_{i=1}^{k}(a_i!e_i, \lambda_i).P_i) \stackrel{def}{=} 0$$

$$c_{a?v}(P|Q) \stackrel{def}{=} c_{a?v}(P) + c_{a?v}(Q)$$

$$c_{a!v}(\textstyle\sum_{i=1}^{k}(a_i?x_i, \lambda_i).P_i) \stackrel{def}{=} 0$$

$$c_{a!v}(\textstyle\sum_{i=1}^{k}(a_i!v_i, \lambda_i).P_i) \stackrel{def}{=} \sum_{i \in \{j | 1 \leq j \leq k, a_j = a, v_j = v\}} \lambda_i$$

$$c_{a!v}(P|Q) \stackrel{def}{=} c_{a!v}(P) + c_{a!v}(Q)$$

Notice that we require e to be already evaluated to v in $c_{a!v}(\sum_{i=1}^{k}(a_i!v_i, \lambda_i).P_i)$. The general form of the transition relation is

$$P \xrightarrow{\ \Gamma\ }_\gamma P'$$

with the usual meaning, where γ is the proof for the transition; in fact, in order to keep track of transition multiplicity in the definition of the CTMC associated to a generic process $P \in \mathcal{C}$, we use *proved* LTSs. Proved transitions have been widely used in the context of formal operational semantics definition techniques for process algebras (see, e.g., [3, 5–7]), including Markovian process algebras [10].

In the context of our Markovian extension of CCS, proofs are finite strings belonging to the set $Prf \stackrel{def}{=} \{+_i, \overset{oL}{|}, \overset{oR}{|}, \overset{\tau\!L}{|}, \overset{\tau\!R}{|}, \overset{\tau L}{|}, \overset{\tau R}{|}, (,)\}^*$ for $i \in \mathbb{N}$. The operational semantics rules are given below:

(IN)
$$\frac{1 \le i \le k,\ v \in V}{\sum_{i=1}^{k}(a_i?x_i,\lambda_i).P_i \xrightarrow{a_i?v,\lambda_i}_{+_i} P_i[v/x_i]}$$

(OUT)
$$\frac{1 \le i \le k}{\sum_{i=1}^{k}(a_i!v_i,\lambda_i).P_i \xrightarrow{a_i!v_i,\lambda_i}_{+_i} P_i}$$

(PoL)
$$\frac{P \xrightarrow{a!v,\lambda_p}_{\gamma_p} P',\ Q \xrightarrow{a?v,\lambda_q}_{\gamma_q} Q'}{P|Q \xrightarrow{\tau(a,v,\lambda_p,\lambda_q,c_{a!v}(P|Q),c_{a?v}(P|Q))}_{(\gamma_p \overset{oL}{\mid} \gamma_q)} P'|Q'}$$

(PoR)
$$\frac{P \xrightarrow{a?v,\lambda_p}_{\gamma_p} P',\ Q \xrightarrow{a!v,\lambda_q}_{\gamma_q} Q'}{P|Q \xrightarrow{\tau(a,v,\lambda_q,\lambda_p,c_{a!v}(P|Q),c_{a?v}(P|Q))}_{(\gamma_p \overset{oR}{\mid} \gamma_q)} P'|Q'}$$

$(I\!\!\not|L)$
$$\frac{P \xrightarrow{\alpha,\lambda}_{\gamma} P'}{P|Q \xrightarrow{\alpha,\lambda}_{\gamma L \mid \gamma} P'|Q}$$

$(I\!\!\not|R)$
$$\frac{Q \xrightarrow{\alpha,\lambda}_{\gamma} Q'}{P|Q \xrightarrow{\alpha,\lambda}_{\gamma R \mid \gamma} P|Q'}$$

$(I\tau L)$
$$\frac{P \xrightarrow{\tau(a,v,\lambda_O,\lambda_I,\mu_{cO},\mu_{cI})}_{\gamma} P'}{P|Q \xrightarrow{\tau(a,v,\lambda_O,\lambda_I,\mu_{cO}+c_{a!v}(Q),\mu_{cI}+c_{a?v}(Q))}_{\tau L \mid \gamma} P'|Q}$$

$(I\tau R)$
$$\frac{Q \xrightarrow{\tau(a,v,\lambda_O,\lambda_I,\mu_{cO},\mu_{cI})}_{\gamma} Q'}{P|Q \xrightarrow{\tau(a,v,\lambda_O,\lambda_I,\mu_{cO}+c_{a!v}(P),\mu_{cI}+c_{a?v}(P))}_{\tau R \mid \gamma} P|Q'}$$

In what follows we say that there is a $a!v$-transition ($a?v$-transition, respectively) from P to Q whenever $P \xrightarrow{a!v,\lambda}_{\gamma} Q$ ($P \xrightarrow{a?v,\lambda}_{\gamma} Q$, respectively), for some λ and γ. Similarly, we say that there is a (a,v)-transition from P to Q whenever $P \xrightarrow{\tau(a,v,\lambda_O,\lambda_I,\mu_{cO},\mu_{cI})}_{\gamma} Q$, for some $\lambda_O, \lambda_I, \mu_{cO}, \mu_{cI}$, and γ. Finally, we say that there is a τ-transition from P to Q if there is a (a,v)-transition from P to Q for some a and v. Below we define the set of derivatives reachable via transitions denoting interactions.

Definition 1 (τ-derivatives).
For set $C \subseteq \mathcal{C}$, the set of τ-derivatives of C, denoted $Der_\tau(C)$, is the smallest set such that:

- $C \subseteq Der_\tau(C)$, and
- if $Q \in Der_\tau(C)$ and $Q \xrightarrow{\tau(a,v,\lambda_O,\lambda_I,\mu_{cO},\mu_{cI})}_\gamma Q'$, for some $a, v, \lambda_O, \lambda_I, \mu_{cO}, \mu_{cI}$, and γ, then also $Q' \in Der_\tau(C)$

The CTMC associated to each closed process term P is defined as follows:

Definition 2 (Semantics).

For closed process P, the CTMC of P is defined as $CTMC[P] \overset{def}{=} (S, \mathcal{R})$ where

- $S \overset{def}{=} Der_\tau(\{P\})$
- *For all $s, s' \in S$,*

$$\mathcal{R}[s, s'] \overset{def}{=} \sum_{s \xrightarrow{\tau(a,v,\lambda_O,\lambda_I,\mu_{cO},\mu_{cI})}_\gamma s'} \frac{\lambda_O}{\mu_{cO}} \cdot \frac{\lambda_I}{\mu_{cI}} \cdot min(\mu_{cO}, \mu_{cI})$$

with $\mathcal{R}[s, s'] = 0$ if there is no τ-transition from s to s'.

In the sequel, for generic CTMC $(S, \mathcal{R})$, $s \in S$, $C \subseteq S$ we let $\mathcal{R}[s, C] \overset{def}{=} \sum_{s' \in C} \mathcal{R}[s, s']$

4 Associativity

In this section we give the proof of the associativity result. We start with a few lemmas.

Lemma 1.
For all $P \in \mathcal{C}$, if $P \xrightarrow{\tau(a,v,\lambda_O,\lambda_I,\mu_{cO},\mu_{cI})}_\gamma P'$ for some $a, v, \lambda_O, \lambda_I, \mu_{cO}, \mu_{cI}, \gamma$ and P', then also $P' \in \mathcal{C}$.

Proof: Trivial, from the relevant definitions. $\square$

An obvious consequence of the above lemma is that $\mathcal{C} = Der_\tau(\mathcal{C})$.

Lemma 2.
For all $P, P' \in \mathcal{C}$: $\sum_{P \xrightarrow{\tau(a,v,\lambda_O,\lambda_I,\mu_{cO},\mu_{cI})}_\gamma P'} \frac{\lambda_O}{\mu_{cO}} \cdot \frac{\lambda_I}{\mu_{cI}} \cdot min(\mu_{cO}, \mu_{cI}) < \infty$

Proof: The assert follows from the fact that there is a *finite* number of τ-transitions from P to P', as it can easily be seen from the definition of the operational semantics, and that all quantities involved in the sum are finite and different from 0. $\square$

As a consequence of the above two lemmas we can consider *the* CTMC of the complete language, namely $CTMC[\mathcal{C}]$ and the notion of strong Markovian bisimulation equivalence $\sim_M$ over closed terms:

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Definition 3.
$CTMC[\mathcal{C}]$ is defined as follows: $CTMC[\mathcal{C}] \stackrel{def}{=} (S, \mathcal{R})$ where

- *$S \stackrel{def}{=} Der_\tau(\mathcal{C})$*
- *For all $s, s' \in S$,*

$$\mathcal{R}[s, s'] \stackrel{def}{=} \sum_{s \xrightarrow{\tau(a, v, \lambda_O, \lambda_I, \mu_{cO}, \mu_{cI})}_\gamma s'} \frac{\lambda_O}{\mu_{cO}} \cdot \frac{\lambda_I}{\mu_{cI}} \cdot min(\mu_{cO}, \mu_{cI})$$

with $\mathcal{R}[s, s'] = 0$ if there is no τ-transition from s to s'.

The following definition is recalled from [4].

Definition 4 (Strong Markovian bisimilarity).
Given generic CTMC $(S, \mathcal{R})$

- *Two states $s, s' \in S$ are strong Markovian bisimilar, written $s \sim_M s'$, if and only if there exists a Markovian bisimulation B_M on S with $(s, s') \in B_M$.*
- *An equivalence relation B_M on S is a Markovian bisimulation on S if and only if for all $(s, s') \in B_M$ and for all $C \in S_{/B_M}$ the following condition holds: $\mathcal{R}[s, C] = \mathcal{R}[s', C]$.*

The associativity result follows:

Theorem 1. *For all $P, Q, R \in \mathcal{C}$ the following holds: $(P|Q)|R \sim_M P|(Q|R)$.*

Proof:
We first define the relation $\doteq$ as the smallest binary relation on $\mathcal{C}$ induced by the following laws; for all $P, Q, R \in \mathcal{C}$:

$P \doteq P$
$P \doteq Q \Rightarrow Q \doteq P$
$P \doteq Q \wedge Q \doteq R \Rightarrow P \doteq R$
$P|(Q|R) \doteq (P|Q)|R$

Clearly, $\doteq$ is an equivalence relation and $\mathcal{C}_{/\doteq}$ is the set below:
$\{\{P|(Q|R), (P|Q)|R\} \mid P, Q, R \in \mathcal{C}\} \cup$
$\{\{P\} \mid P \in \mathcal{C}, \nexists P', Q', R' \in \mathcal{C} : P = P'|(Q'|R') \text{ or } P = (P'|Q')|R'\}$
    We show now that $\doteq$ is a strong Markovian bisimulation equivalence.
    Take generic $s, s' \in \mathcal{C}$ with $s \doteq s'$. We have to show that $\mathcal{R}[s, C] = \mathcal{R}[s', C]$ for all $C \in \mathcal{C}_{/\doteq}$. The only non-trivial case is for $s = P|(Q|R)$ and $s' = (P|Q)|R$, for some $P, Q, R \in \mathcal{C}$. We have to show that $\mathcal{R}[P|(Q|R), C] = \mathcal{R}[(P|Q)|R, C]$, for all $P, Q, R \in \mathcal{C}$ and $C \in \mathcal{C}_{/\doteq}$. The derivation proceeds as follows, where two auxiliary lemmas are used which are proved later on:

$\mathcal{R}[(P|Q)|R, C]$

$=$ {Lemma 3 below, def. of $\mathcal{R}[s,C]$}

$\sum_{(P'|Q')|R'\in C} \mathcal{R}[(P|Q)|R, (P'|Q')|R']$

$=$ {Lemma 4 below}

$\sum_{P'|(Q'|R')\in C} \mathcal{R}[P|(Q|R), P'|(Q'|R')]$

$=$ {Lemma 3 below, def. of $\mathcal{R}[s,C]$}

$\mathcal{R}[P|(Q|R), C].$ $\square$

Lemma 3. *For all $P, Q, R, s \in \mathcal{C}$, the following holds:*

i) *if there is a τ-transition from $(P|Q)|R$ to s, then $s = (P'|Q')|R'$ for some $P', Q', R' \in \mathcal{C}$, and*

ii) *if there is a τ-transition from $P|(Q|R)$ to s, then $s = P'|(Q'|R')$ for some $P', Q', R' \in \mathcal{C}$.*

Proof:
The assert follows directly from the definition of the operational semantics. $\square$

Lemma 4. *For all $P, Q, R, P', Q', R' \in \mathcal{C}$, the following holds:*
$\mathcal{R}[(P|Q)|R, (P'|Q')|R'] = \mathcal{R}[P|(Q|R), P'|(Q'|R')]$

Proof:
The assert follows directly from Lemma 5 and the definition of $\mathcal{R}[.,.]$. $\square$

Lemma 5. *For all $P, Q, R \in \mathcal{C}$, the following holds:*

i) *for all $P', Q', R' \in \mathcal{C}$, $\lambda_O, \lambda_I, \mu_{cO}, \mu_{cI}$:*
there exists γ_1 s.t. $P|(Q|R) \xrightarrow{\tau(a,v,\lambda_O,\lambda_I,\mu_{cO},\mu_{cI})}_{\gamma_1} P'|(Q'|R')$ if and only if
there exists γ_2 s.t. $(P|Q)|R \xrightarrow{\tau(a,v,\lambda_O,\lambda_I,\mu_{cO},\mu_{cI})}_{\gamma_2} (P'|Q')|R';$

ii) *the number of outgoing τ-transitions of $(P|Q)R$ is the same as the number of outgoing τ-transitions of $P|(Q|R)$.*

Proof:
We first prove Point (i) of the lemma. We only consider the direct implication explicitly; the proof for the reverse implication is similar. Suppose therefore that $P|(Q|R) \xrightarrow{\tau(a,v,\lambda_O,\lambda_I,\mu_{cO},\mu_{cI})}_{\gamma_1} P'|(Q'|R')$. There are several cases to be considered:

1. the (a,v)-transition of $P|(Q|R)$ originates from an (a,v)-transition of P;
2. the (a,v)-transition of $P|(Q|R)$ originates from an interaction of P and Q;
3. the (a,v)-transition of $P|(Q|R)$ originates from an interaction of Q and R;
4. the (a,v)-transition of $P|(Q|R)$ originates from an interaction of P and R;
5. the (a,v)-transition of $P|(Q|R)$ originates from an (a,v)-transition of Q;
6. the (a,v)-transition of $P|(Q|R)$ originates from an (a,v)-transition of R.

We show the proof for cases 1,2, and 3 only, since case 4 is similar to case 2, and cases 5 and 6 are similar to case 1.

Case 1: *the (a,v)-transition of $P|(Q|R)$ originates from an (a,v)-transition of P*. In this case, we know that $Q' = Q$ and $R' = R$. Moreover, from the operational semantics definition, we also know that:

$$\mu_{cO} = \mu_{cO'} + c_{a!v}(Q|R)$$

$$\mu_{cI} = \mu_{cI'} + c_{a?v}(Q|R)$$

and that

$$P \xrightarrow{\tau(a,v,\lambda_O,\lambda_I,\mu_{cO'},\mu_{cI'})}_\gamma P'$$

with $\gamma_1 = \overset{\tau L}{|}\ \gamma$, for some γ. Clearly, from this transition of P we can deduce a (a,v)-transition of $(P|Q)|R$ as follows:

$$\frac{\dfrac{P \xrightarrow{\tau(a,v,\lambda_O,\lambda_I,\mu_{cO'},\mu_{cI'})}_\gamma P'}{P|Q \xrightarrow{\tau(a,v,\lambda_O,\lambda_I,\mu_{cO'}+c_{a!v}(Q),\mu_{cI'}+c_{a?v}(Q))}_{\overset{\tau L}{\underset{|\ \gamma}{}}} P'|Q}[I\tau L]}{(P|Q)|R \xrightarrow{\tau(a,v,\lambda_O,\lambda_I,\mu_{cO'}+c_{a!v}(Q)+c_{a!v}(R),\mu_{cI'}+c_{a?v}(Q)+c_{a?v}(R))}_{\overset{\tau L \tau L}{\underset{|\ |\ \gamma}{}}} (P'|Q)|R}[I\tau L]$$

and the assert is proven with $\gamma_2 = \overset{\tau L \tau L}{|\ |}\ \gamma$, noting that $c_{a!v}(Q) + c_{a!v}(R) = c_{a!v}(Q|R)$ and $c_{a?v}(Q) + c_{a?v}(R) = c_{a?v}(Q|R)$ by definition of $c_{a!v}$ and $c_{a?v}$.

Case 2: *the (a,v)-transition of $P|(Q|R)$ originates from an interaction of P and Q*. W.l.g. let us assume that the synchronisation involves an output $a!v$-transition of P and an input $a?v$-transition of Q, while $R = R'$. From the definition of the operational semantics, we know that $P \xrightarrow{a!v,\lambda_O}_{\gamma_p} P'$, $Q \xrightarrow{a?v,\lambda_I}_{\gamma_q} Q'$, and $(Q|R) \xrightarrow{a?v,\lambda_I}_{\overset{\tau L}{\underset{|\ \gamma_q}{}}} (Q'|R)$ for some γ_p and γ_q and that $\mu_{cO} = c_{a!v}(P|(Q|R))$, $\mu_{cI} = c_{a?v}(P|(Q|R))$. Clearly, from the transitions of P and Q, we can deduce an (a,v)-transition of $(P|Q)|R$ as follows:

$$\frac{\dfrac{P \xrightarrow{a!v,\lambda_O}_{\gamma_p} P', Q \xrightarrow{a?v,\lambda_I}_{\gamma_q} Q'}{P|Q \xrightarrow{\tau(a,v,\lambda_O,\lambda_I,c_{a!v}(P|Q),c_{a?v}(P|Q))}_{\overset{oL}{\underset{(\gamma_p\ |\ \gamma_q)}{}}} P'|Q'}[PoL]}{(P|Q)|R \xrightarrow{\tau(a,v,\lambda_O,\lambda_I,c_{a!v}(P|Q)+c_{a!v}(R),c_{a?v}(P|Q)+c_{a?v}(R))}_{\overset{\tau L \quad oL}{\underset{|\ (\gamma_p\ |\ \gamma_q)}{}}} (P'|Q')|R}[I\tau L]$$

and the assert is proven with $\gamma_2 = \overset{\tau L}{|}\ (\gamma_p \overset{oL}{|}\ \gamma_q)$, noting that $c_{a!v}(P|(Q|R)) = c_{a!v}(P) + c_{a!v}(Q) + c_{a!v}(R) = c_{a!v}(P|Q) + c_{a!v}(R)$ and similarly $c_{a?v}(P|(Q|R)) = c_{a?v}(P) + c_{a?v}(Q) + c_{a?v}(R) = c_{a?v}(P|Q) + c_{a?v}(R)$.

Case 3: *the (a,v)-transition of $P|(Q|R)$ originates from an interaction of Q and R*. W.l.g. let us assume that the synchronisation involves an output $a!v$-transition

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of Q and an input $a?v$-transition of R, while $P = P'$. From the definition of the operational semantics, we know that $Q \xrightarrow{a!v,\lambda_O}_{\gamma_q} Q'$, $R \xrightarrow{a?v,\lambda_I}_{\gamma_r} R'$ for some γ_q and γ_r and that $\mu_{cO} = c_{a!v}(Q|R) + c_{a!v}(P)$, and $\mu_{cI} = c_{a?v}(Q|R) + c_{a?v}(P)$. Clearly, from the transitions of R and Q, we can deduce an (a,v)-transition of $(P|Q)|R$ as follows:

$$\cfrac{\cfrac{Q \xrightarrow{a!v,\lambda_O}_{\gamma_q} Q'}{P|Q \xrightarrow{a!v,\lambda_O}_{\substack{\not{\gamma}R \\ |\ \gamma_q}} P|Q'}[I\not{\vdash}R],\ R \xrightarrow{a?v,\lambda_I}_{\gamma_r} R'}{(P|Q)|R \xrightarrow{\tau(a!v,\lambda_O,\lambda_I,r_{a!v}((P|Q)|R),r_{a?v}((P|Q)|R))}_{\substack{\not{\gamma}R\quad oL \\ (\ |\ \gamma_q\ |\ \gamma_r)}} (P|Q')|R'}[PoL]$$

and the assert is proven with $\gamma_2 = (\ |\ \overset{\not{\gamma}R}{\gamma_q}\ |\ \overset{oL}{\gamma_r})$, noting that $c_{a!v}((P|Q)|R) = c_{a!v}(P) + c_{a!v}(Q|R)$, and similarly $c_{a?v}((P|Q)|R) = c_{a?v}(P) + c_{a!v}(Q|R)$.

We now move to the proof of point (ii) of the lemma, which follows essentially the same pattern as that for point (i). For generic a and v, let n be the number of (a,v)-transitions originating from $(P|Q)|R$ and, similarly let m be the number of (a,v)-transitions originating from $P|(Q|R)$. We have to show that $n = m$. Clearly, $n = n_p + n_q + n_r + n_{pq} + n_{pr} + n_{qr}$, where

- n_p is the number of (a,v)-transitions of $(P|Q)|R$, originating from (a,v)-transitions of P;
- n_q is the number of (a,v)-transitions of $(P|Q)|R$, originating from (a,v)-transitions of Q;
- n_r is the number of (a,v)-transitions of $(P|Q)|R$, originating from (a,v)-transitions of R;
- n_{pq} is the number of (a,v)-transitions of $(P|Q)|R$, originating from (a,v)-interactions between P and Q;
- n_{pr} is the number of (a,v)-transitions of $(P|Q)|R$, originating from (a,v)-interactions between P and R;
- n_{qr} is the number of (a,v)-transitions of $(P|Q)|R$, originating from (a,v)-interactions between Q and R

and similarly for $m_p + m_q + m_r + m_{pq} + m_{pr} + m_{qr} = m$. We show that $n_p = m_p$ and that $n_{pq} = m_{pq}$ since all other cases are similar. Let k_p be the number of (a,v)-transitions of P. Each such transition, generates *exactly one* (a,v)-transition in $(P|Q)|R$ and *exactly one* (a,v)-transition in $P|(Q|R)$ according to the following derivations:

$$\cfrac{\cfrac{P \xrightarrow{\tau(a,v,\lambda_O,\lambda_I,\mu_{cO},\mu_{cI})}_{\gamma} P'}{P|Q \xrightarrow{\tau(a,v,\lambda_O,\lambda_I,\mu_{cO}+c_{a!v}(Q),\mu_{cI}+c_{a?v}(Q))}_{\substack{\tau L \\ |\ \gamma}} P'|Q}[I\tau L]}{(P|Q)|R \xrightarrow{\tau(a,v,\lambda_O,\lambda_I,\mu_{cO}+c_{a!v}(Q)+c_{a!v}(R),\mu_{cI}+c_{a?v}(Q)+c_{a?v}(R))}_{\substack{\tau L \tau L \\ |\ |\ \gamma}} (P'|Q)|R}[I\tau L]$$

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and

$$\frac{P \xrightarrow{\tau(a,v,\lambda_O,\lambda_I,\mu_{cO},\mu_{cI})}_\gamma P'}{P|(Q|R) \xrightarrow{\tau(a,v,\lambda_O,\lambda_I,\mu_{cO}+c_{a!v}(Q|R),\mu_{cI}+c_{a?v}(Q|R))}_{\substack{\tau L \\ |\ \gamma}} P'|(Q|R)}[I\tau L]$$

So, for each (a,v)-transition of P, with proof, say, γ there is exactly one (a,v)-transition of $(P|Q)|R$, with proof $\overset{\tau L \tau L}{|\ |}\ \gamma$, and exactly one (a,v)-transition of $P|(Q|R)$, with proof $\overset{\tau L}{|}\ \gamma$; thus, $n_p = k_p = m_p$.

We now show that $n_{pq} = m_{pq}$. Each (a,v)-transition in $(P|Q)|R$ generated from an interaction between P and Q requires an output $a!v$-transition in one component, say P w.l.g., and a corresponding input $a?v$-transition in the other component (Q). Each pair of such transitions generates a *unique* (a,v)-transition in $(P|Q)|R$, e.g.

$$\frac{P \xrightarrow{a!v,\lambda_O}_{\gamma_p} P', Q \xrightarrow{a?v,\lambda_I}_{\gamma_q} Q'}{\dfrac{P|Q \xrightarrow{\tau(a,v,\lambda_O,\lambda_I,c_{a!v}(P|Q),c_{a?v}(P|Q))}_{\substack{oL \\ (\gamma_p\ |\ \gamma_q)}} P'|Q'}{(P|Q)|R \xrightarrow{\tau(a,v,\lambda_O,\lambda_I,c_{a!v}(P|Q)+c_{a!v}(R),c_{a?v}(P|Q)+c_{a?v}(R))}_{\substack{\tau L\quad oL \\ |\ (\gamma_p\ |\ \gamma_q)}} (P'|Q')|R}[I\tau L]}[PoL]$$

and, similarly, a *unique* (a,v)-transition in $P|(Q|R)$, as follows:

$$\frac{P \xrightarrow{a!v,\lambda_O}_{\gamma_p} P', \dfrac{Q \xrightarrow{a?v,\lambda_I}_{\gamma_q} Q'}{Q|R \xrightarrow{a?v,\lambda_I}_{\substack{\tau' L \\ |\ \gamma_q}} Q'|R}[I\nmid L]}{P|(Q|R) \xrightarrow{\tau(a!v,\lambda_O,\lambda_I,c_{a!v}(P|(Q|R)),c_{a?v}(P|(Q|R)))}_{\substack{\tau' L\quad oL \\ (\ |\ \gamma_q\ |\ \gamma_r)}} P'|(Q')|R)}[PoL]$$

Thus $n_{pq} = m_{pq}$. □

5 Conclusions

Stochastic extension of process algebras based on a two-party, CCS-like, synchronisation paradigm, with rates associated to actions, requires the explicit treatment of communication polarity. We illustrated why a straightforward application of the apparent rate approach developed by Hillston for multi-party, CSP-like, synchronisation paradigm is not satisfactory in a CCS-like interaction framework; in fact such an approach does not permit preservation of properties like associativity of important operators. We showed how the approach can be adapted and we provided a formal SOS for a simple dialect of value passing CCS, based on proved labelled transition systems. We finally showed how our approach leads to semantics that preserve associativity: more precisely we proved that the CTMC of $P|(Q|R)$ is Strong Markovian Bisimilar to the CTMC of $(P|Q)|R$.

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