Nonlinear model identification and seethrough cancellation from recto-verso data

Emanuele Salerno · **Francesca Martinelli** · **Anna Tonazzini**

Abstract The problem of seethrough cancellation in digital images of double-sided documents is addressed. Previous approaches to solve this problem from recto-verso pairs of grayscale data images show a number of drawbacks, ranging from errors due to an inadequate data model to excessive computational complexities. While satisfying the need to assume a nonlinear convolutional mixture model and to estimate its parameters along with the recto and verso patterns, we propose a simple and fast strategy to estimate the trasparency of the paper and the seethrough convolutional kernel, thus enabling an efficient correction of this distortion. Compared to other separation strategies, our choice is slightly more cumbersome since average background values must be estimated and a pure showthrough area must be isolated manually by the operator. Although the procedure cannot be fully automatic, however, it outperforms other restoration strategies, especially if based on linear instantaneous models.

Keywords Document image processing · Seethrough cancellation · Nonlinear image models

1 Introduction

Digital images of documents are often affected by distortions originating from several causes. The patterns that carry the relevant information in the digital object can be affected by interferences depending on the state of conservation of the physical document. As an example, in an image of a paper document, one relevant pattern could be a text printed or handwritten on it, and the interfering patterns could be defects in the document support (the paper), such as tears, mold or stains. Another source of distortion is the digitization equipment, which introduces sensor noise and other artifacts due to its finite optical resolution and other optical aberrations. Further problems cen emerge from the interaction between the document and the light used to probe it.

In this paper, we are concerned with a kind of distortion that affects double-sided documents and can arise from both the physical status of the original and the image capture process. This is the *seethrough*, that is, the interference produced on one side by some pattern belonging to the opposite side. This can be caused by the transparency of the support combined with the illumination conditions (*show-through*), or by bleeding of ink through the support (*bleed-through*). One of the objectives of document image processing is to improve the readability of the originals by mitigating the effects of interferences such as seethrough, or helping the reader to distinguish between the different patterns appearing in the document. Whereas a poor accomplishment of this task can be of moderate importance to a human reader, who is by nature a very skilled pattern recognizer, it becomes extremely important when an automatic procedure (optical character recognition, and the like) is supposed to read the document. Indeed, it is demonstrated in [19] that an effective seethrough removal can highly increase the performance of a commercial character recognition application on a printed document. For heavily distorted texts, a digital restoration can also be a prerequisite to any further low-level or high-level processing. More generally, mitigating distortions is often an essential step towards segmentation, which is "the key step towards high-level vision modeling and analysis,

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including object characterization, detection, and classification" [16].

Any restoration strategy must consider a data model, be it empirical or based on a physical analysis of the distortion. Introducing such a model is a way to enforce constraints on the problem to be solved, which is always an inverse problem and needs regularization. Constraints can be posed on either some properties of the original patterns or the generation of the observed data (in our case, the process that causes interference between different patterns). In the former approach, seethrough removal can be seen as a classification problem, that is, the appearance map of one side of the document can be partitioned in several classes (*e.g.*, "background", "main text" and "seethrough"), and the areas classified as seethrough can be removed or inpainted by using the features of the background class [13,22]. In [1], the authors exploit the fact that the small-scale features of the interfering strokes are less intense than the ones of the foreground pattern. The result is a very fast, local, and noniterative procedure that provides results comparable to the ones obtained by other strategies. In [5], the data come from both sides of the document page, the classification is based on a user-input training set, and helped by a dual-layer Markov random field model that enforces both intra-side and inter-side constraints. An example of the alternative strategy, the one where the model constrains data generation, is to consider the page appearance as a linear superposition of the front-page (*recto*) and the back-page (*verso*) patterns [19]. It is easy to see that, under many respects, this model is too simplistic. Indeed, any text stroke in the recto simply covers the possible seethrough, thus producing an appearance that only depends on the front pattern. In general, as the recto becomes darker, the seethrough contribution to the page appearance becomes weaker. Seethrough is thus a nonlinear effect depending on both the front and the back patterns. Besides being nonlinear, it should also be considered noninstantaneous since, for the diffusion of light inside the support, seethrough is both an attenuated and a smoothed copy of the verso pattern. As a result, any restoration technique based on a linear and instantaneous model is affected by artifacts. In particular, the linearity assumption often produces unwanted splits in the recto characters, where they overlap the seethrough. The instantaneousness assumption in linear recto-verso models, in turn, seldom produces a good cancellation, since more or less pronounced halos can remain around the zones where the seethrough has been removed. To reduce these effects, a linear noninstantaneous model is proposed in [21], with unknown point spread functions (*psf*) evaluated by a Bayesian strategy with appropriate priors. In [11], a nonlinear diffusion model of bleed-through is introduced, based on the physical mechanism of liquid seepage in porous media. Nonlinear models have also been proposed in [15] and [10].

As far as the practical restoration strategies are concerned, we are interested in the ones that assume generative models whose parameters are unknown or partially unknown. These models can account for observed data coming from either the recto alone or both sides of the document, and consisting of either grayscale or color/multispectral quantities. In general, these modalities and any other possible diversity acquisition strategies are said to form a *multichannel* data set. If the original patterns are allowed to be partially or totally overlapped to one another, restoration can be seen as a blind source separation problem. From multichannel data and a linear model, the problem can be solved by linear independent component analysis [7,17]. The application to color or multispectral one-sided acquisitions is reported in [18]. Results from two-sided acquisitions are reported in [19] (grayscale data) and [20] (color-multispectral data). Despite the limitations of a linear data model, these techniques proved to be useful in many practical cases. The nonlinear approach introduced in [15] is based on a physical model of the transmission of light through the paper and is valid for scanned images. The diffusion of light is accounted for by a *show-through psf*, which is then estimated adaptively in a restoration scheme where the scanned version of the verso is used as an approximation of the original verso pattern. This scheme can work if estimates of the background levels are available, and a suitable threshold is established to identify pure showthrough pixels. Both these requirements rely on human operators. Exploiting the same model, with known nonlinearity and unknown psf, a restoration based on the optimization of a total-variation-regularized cost function is proposed in [14]. In [10], a nonlinear (instantaneous) model is built, and a clever experiment is designed to estimate empirically the type of nonlinearity, which is then given a parametric form to be estimated by a recursive network. A predetermined psf is eventually included in the separating network to correct some of the unwanted effects of the instantaneous model.

The linear methods can be totally blind and automatic but, being based on unphysical assumptions, are often affected by strong model errors. The physically based models, on the other hand, are often too specific. The model in [15] is specifically related to image capture by a (flatbed) scanner with white backing. Its use in a nonblind scheme, that is, one that replaces the original patterns by the scanned recto and verso, results in an erosion of the characters. Digital document images are not always produced by scanners, and can come from ancient documents where it is difficult to establish a meaningful average background level. Working with ancient documents also means to cope with additional disturbances, on which adaptive filtering can become unstable or unreliable. Some of these drawbacks can also affect the approach in [10], when applied to ancient documents, since its nonlinearity is based on a preliminary calibration that depends on the features of the support material. All the nonlinear and iterative approaches, finally, must cope with computational complexity issues and, sometimes, with existence and uniqueness problems (see for example [4,6]).

Assuming the model proposed in [15], we describe here a strategy to avoid some of these drawbacks, yet maintaining an acceptable computational cost. Since we plan to use the model in a more general setting, its physical significance is not central to us. Thus, the estimated model parameters will only be justified on experimental grounds, even if they violate some of the original assumptions. To make the method stable against a number of possible interferences, rather than using an adaptive filter we rely on an offline estimation of the *seethrough psf*, based on a manual selection of an area where only background and seethrough are present. The model thus built is then used to implement an iterative but fast restoration algorithm that, after a reduced number of iterations, removes almost completely the residual halos and reduces the erosion effect on the restored characters. Also, by exploiting the seethrough psf, we are allowed to recover possible small errors in recto-verso registration, a preliminary step that is always needed in appearance-based multisource image analysis. Under a computational point of view we observe that, since we do not need to estimate the psf at each iteration, the overall procedure is not too expensive. As happens with the algorithm in [15], our approach requires a human operator to identify pure background areas in both the recto and the verso. The operator is also requested to find an isolated pure seethrough area, but this is not a difficult task, since it is not necessary to establish any threshold. Another limitation is that our strategy is strictly valid if both the average background levels and the seethrough psf are stationary throughout the image. This can be mitigated by dividing the data support into compact subsets where the stationarity requirement is approximately satisfied, or by some strategy that allows the transparency of the support to be estimated pixel by pixel. Some hints in this sense can be found in Section 2.

This paper is organized as follows. Section 2 recalls the features of the model and describes a strategy to estimate the seethrough psf. Section 3 details the iterative restoration strategy we adopt once the psf has been estimated. Section 4 shows some experimental results to validate both the model and the algorithm. Section 5 concludes the paper.

2 Offline seethrough estimation for a nonlinear mixture model

As anticipated, our model is the one proposed in [15] for show-through cancellation. We briefly describe here the form of interest to us. Let us assume the existence of pure recto and verso patterns, represented by the reflectance maps $x_r(t)$ and $x_v(t)$, respectively, where t is the position vector. We model the recto and verso appearances as the vector map $\mathbf{x}^{obs}(t) = [x_r^{obs}(t), x_v^{obs}(t)]^T = \mathbf{f}(x_r(t), x_v(t))$, where $\mathbf{f} = [f_{rv}, f_{vr}]^T$ is generally a nonlinear and noninstantaneous vector function of both the pure patterns. The appearances can also be described through the density maps, $D_k(t)$ (where k is r or v), which are related to the reflectances by the following pair of transformations

$$D_k(t) = -\ln \frac{x_k(t)}{N_k}, \qquad \frac{x_k(t)}{N_k} = e^{-D_k(t)} \tag{1}$$

where N_k denote the average background reflectances, that is, the mean reflectances of the recto and verso regions where no text or other patterns are present. Note that the density vanishes where the reflectance equals N_k, assumes negative values where the reflectance is greater than N_k, and goes to positive infinity where the reflectance goes to zero (*i.e.*, in the black pixels). In practice, to avoid the latter situation, we set all the black pixels at a small positive value. In terms of densities, the following mixture model is proposed in [15]

$$\begin{aligned} D_r^{obs} &= D_r + q h_{rv} * (1 - e^{-D_v}) \\ D_v^{obs} &= D_v + q h_{vr} * (1 - e^{-D_r}) \end{aligned} \tag{2}$$

where the asterisk means convolution, q is a real nonnegative parameter that we call *transparency*, and the seethrough psfs, h_{rv} and h_{vr}, model the noninstantaneousness of the system. To avoid indeterminacies in q, we assume a normalization condition on both the psfs:

$$\int h_{rv,vr}(t)dt = 1 \tag{3}$$

where the integration domain is the entire image. As done in Eq. (2), hereafter we do not make explicit the dependence on position, unless strictly needed. In any case, it is to bear in mind that each position vector in both the observed maps must point to the same spatial location. In our case, this can be featured either by the particular acquisition hardware or by spatial registration of the two data images. We do not give details on registration here (see [20]) but Eq. (2) shows that, once registration is accomplished, the observed recto and verso density maps are nonlinear superpositions of the recto and verso pattern densities with interference maps smoothed by the kernels h_{rv} and h_{vr}. From this model, it can be seen that the observed maps are superpositions of the pure patterns only if the recto or the verso are flipped horizontally. In this paper we take the recto as our reference, thus D_v^{obs} and D_v are the verso density maps after horizontal flipping. From equation (1), to evaluate the densities from the reflectances one needs the values N_r and N_v, which can be evaluated from areas, selected manually, where no other pattern than background is present. The unknowns to be estimated to extract the pure recto and verso patterns are the transparency q and the psfs h_{rv} and h_{vr}. If the interference terms are computed by using D_r^{obs} and D_v^{obs} rather than D_r and D_v, Eq. (2) can easily be solved for q, h_{rv} and h_{vr}, for example by adaptive filtering as done in [15]. We already mentioned some of the drawbacks of this procedure. To avoid them, we chose to leave model (2) as is, without approximating the interfering patterns with the observed maps. Then, an iterative solution can be envisaged, with two alternative options: at each iteration, we can either update all the quantities q, h_{rv}, h_{vr}, D_r and D_v, or just D_r and D_v, once q, h_{rv} and h_{vr} have been estimated once for all. We have been investigating both options. The former, however, is the subject of a separate work [9] and is not treated here. In this paper, we consider the case where only D_r and D_v are estimated iteratively, as a fast alternative to the completely iterative approach. We are planning to compare the relative advantages and disadvantages in the near future. Besides selecting pure background areas to estimate N_r and N_v, we also need to estimate q, h_{rv} and h_{vr} offline, and this requires the identification of some area where pure seethrough is present. Let us first make an important consideration. Normally, there is no reason to assume an anisotropic diffusion of light in the document support. It is thus reasonable to assume $h_{rv}(t) = h_{vr}(t)$. This would lead to try to estimate a single psf, $h(t)$. However, let us admit the existence of a small translational registration error, $t_0 = (i_0, j_0)$, between D_r^{obs} and D_v^{obs}. In this case, if $h(t)$ is the seethrough psf for perfect registration, then we have $h_{rv}(t) = h(i+i_0, j+j_0)$, and $h_{vr}(t) = h(i-i_0, j-j_0)$. This relationship allows us to estimate the psf on one side and derive the psf on the other side by just translating it[1]. For this reason, we drop the subscripts r and v and assume to be able to identify a pure showthrough area in the recto side, that is, an area where D_r is zero, D_v is not identically zero, and where, from (2), we have

$$D_r^{obs} = q \cdot h * (1 - e^{-D_v^{obs}}) \tag{4}$$

Equation (4) allows qh to be estimated by constrained least squares:

$$\widehat{qh} = \arg\min_{qh} \left[\left\| D_r^{obs} - qh * (1 - e^{-D_v^{obs}}) \right\|^2 + \lambda \left\| Lqh \right\|^2 \right] \tag{5}$$

The first squared norm in brackets is a data fit term, whereas the second squared norm constrains the solution to be regular to some degree, identified by operator L, and parameter λ weighs the influences of data fit and regularity on the solution. Equation (5) can easily be solved for qh, choosing λ so that the residual square norm equals the energy of the noise [2,8]. What we need is thus the noise power and a suitable regularization operator L. As the noise power, we assume the variance of the recto background reflectance around its mean N_r. As the regularizing operator, we choose the Laplacian, which enforces second-order smoothness in the solution. The support of qh is initially assumed to coincide with the selected seethrough area. Once the estimation is complete, we could use the nomalization condition (3) to evaluate the transparency q. Actually, the estimated qh normally assumes significant values in a restricted area around its maximum and, since positivity is not enforced, it could also assume physically meaningless negative values. Thus, we compute the transparency after restricting appropriately the support of h and truncating all its negative values. From (4), it is also immediate to note that, in each seethrough pixel, we have

$$q = \frac{D_r^{obs}}{h * (1 - e^{-D_v^{obs}})} \tag{6}$$

which gives us another tool to estimate q once h is available. The possibility of evaluating the registration error t_0 along with h relaxes the requirements on registration, since a small translational misalignment between recto and verso can be compensated.

3 Iterating recto and verso restoration

From (2), we obtain

$$\begin{aligned} D_r &= \max\left\{ \left[D_r^{obs} - q \cdot h_{rv} * (1 - e^{-D_v}) \right], D_r^{min} \right\} \\ D_v &= \max\left\{ \left[D_v^{obs} - q \cdot h_{vr} * (1 - e^{-D_r}) \right], D_v^{min} \right\} \end{aligned} \tag{7}$$

where, of course, the introduction of the lower bounds

$$D_{r,v}^{min} = -\ln \frac{x_{r,v}^{max}}{N_{r,v}} \tag{8}$$

(see also Eq. 1) has no effect if D_r, D_v, q and $h_{rv,vr}$ assume their true values. Once suitable estimates of q and $h_{rv,vr}$ are available, Eq. (7) can provide a number of iterative schemes to estimate D_r and D_v (see also [3]). For example, set $D_r^{(0)} = D_r^{obs}$ and $D_v^{(0)} = D_v^{obs}$, then, for $n = 1, 2, \ldots$

$$\begin{aligned} D_r^{(n)} &= \max\left\{ \left[D_r^{obs} - q \cdot h_{rv} * (1 - e^{-D_v^{(n-1)}}) \right], D_r^{min} \right\} \\ D_v^{(n)} &= \max\left\{ \left[D_v^{obs} - q \cdot h_{vr} * (1 - e^{-D_r^{(n-1)}}) \right], D_v^{min} \right\} \end{aligned} \tag{9}$$

The presence of the lower bounds D_r^{min} and D_v^{min} now prevents the solution from going out of the allowed density range during the iteration. Note, however, that their true values are not known, since they belong to the pure recto and verso patterns, which are not known. One possibility is to use the observed values instead. Alternatively, in the cases where the background inhomogeneities are mainly due to noise or local color variations, they can also be set to different values (for example, zero, i.e. the average background

[1] Since the psf for perfect registration has a peak in $(0,0)$, (i_0, j_0) can easily be evaluated as the offset of the estimated psf with respect to $(0,0)$.

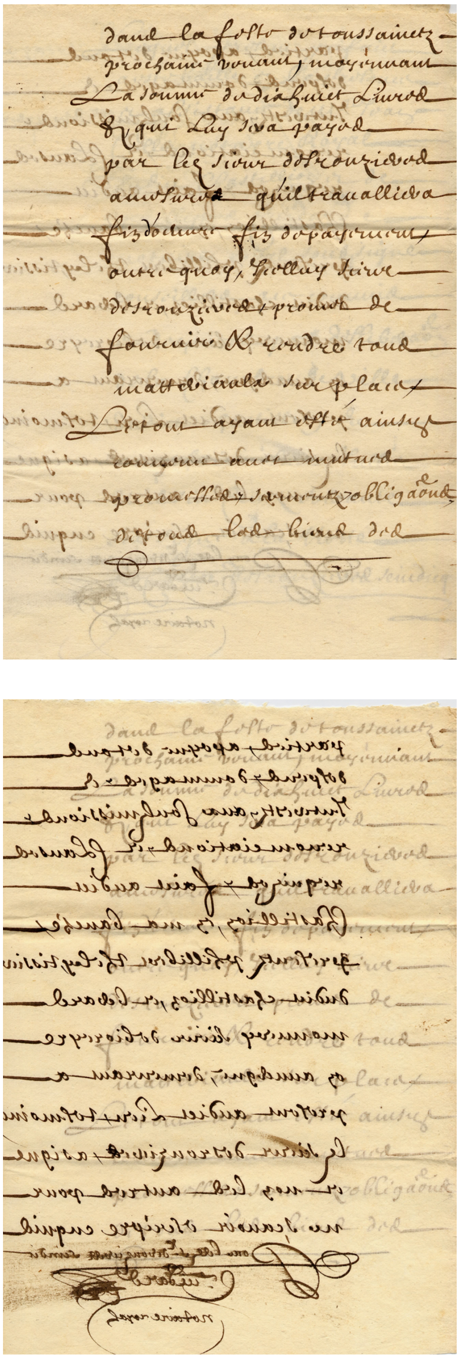

Fig. 1 Double-sided RGB document scan. Top: recto. Bottom: verso

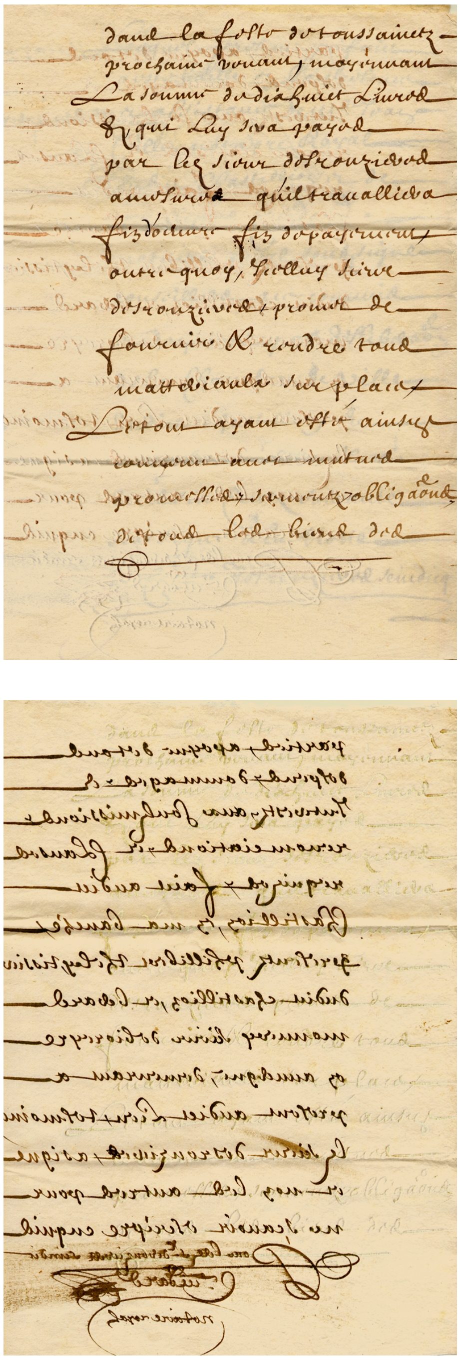

Fig. 2 Outputs of a linear instantaneous restoration algorithm applied to the data images in Fig. 1

value). The result is a restored document whose background is much more uniform than in the observed image. In general, the final solution (if any) may depend on the initial guess. Indeed, we have seen that the map (7) could not have a unique fixed point, depending on the value of q. Conversely, although we have not been able to evaluate it yet, it seems that there exists a maximum q that ensures this property to hold true. The limiting case $q = 0$ always yields the trivial solution $D_{r,v} = D^{obs}_{r,v}$, which, however, does not belong to the feasible solution set since q is estimated offline and fixed. Of course, estimating $q = 0$ would mean no seethrough (or, equivalently, infinitely opaque paper). So far, our experimentation of the scheme (9), with the mentioned initial guess, has been producing good results in nearly all the cases, also with artificial test images constructed so to have perfectly transparent paper.

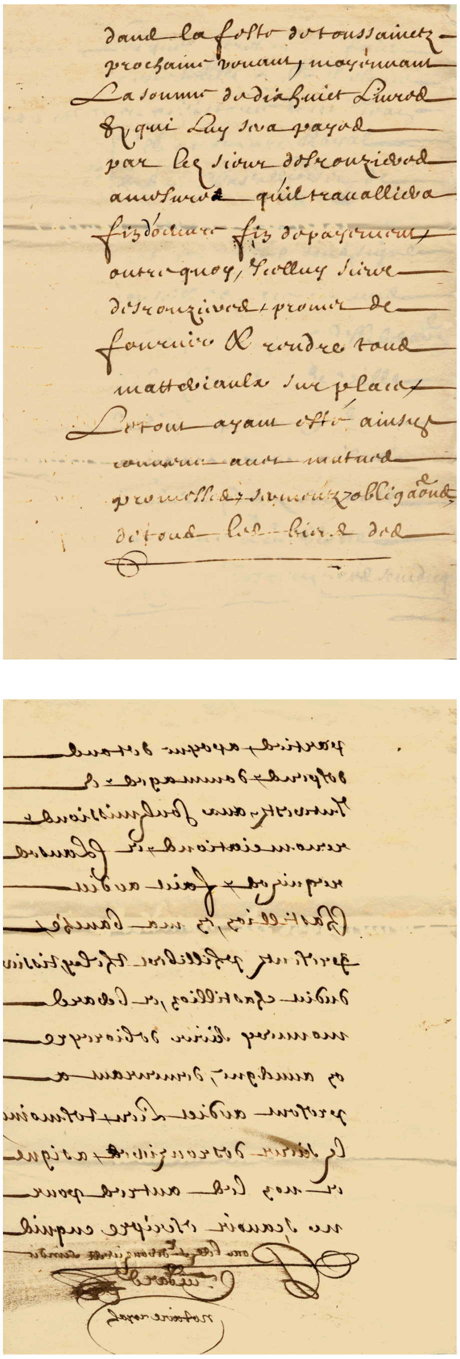

Fig. 3 Data images in Fig. 1 processed by the iterative procedure presented here

4 Validation and experimental results

We tested extensively the restoration scheme (9) on simulated and real data images. The example cases reported here are all real, and are intended to demonstrate qualitatively the effectiveness of the strategy we are proposing. Note that all the relations presented in Sections 2 and 3 are applied to scalar images. However, applying the restoration procedure separately for each channel of an RGB image allows us to recover the original document colors by just composing the three output images. This is the case in two of the examples shown here. In all the figures, the verso sides of the documents are flipped horizontally and spatially registered.

The first example is concerned with a paper manuscript affected by a significant seethrough, Fig. 1, and is intended to compare the results obtained through the technique proposed here and the ones coming from linear instantaneous blind source separation as suggested in [19]. The linear instantaneous results are shown in Fig. 2. Note that all the drawbacks mentioned above are quite apparent. First of all, there is a significant seethrough residual. Moreover, the instantaneousness of the model and some residual misalignment between recto and verso produce halos around the removed strokes, as is apparent, *e.g.*, in the lower part of the restored recto. There are patterns not coming from the verso side but appearing by transparency from the next manuscript page. These can be seen as bluish strokes in the recto and, since they are not accounted for in the model, cannot be removed by the technique considered here, at least when the procedure is applied to the three color channels separately. If unwanted, these patterns can be removed by processing the color image as described in [18]. The result obtained by the technique proposed here on the same data is shown in Fig. 3. The advantages of this technique over the linear one are apparent, as no halo is present anymore. Also in this case, however, there are residual interferences due to either non-seethrough patterns or nonstationarity.

Figure 4 shows a case where the manuscript is affected by heavy distortions, as the interferences between recto and verso are due to both show-through and bleed-through, and the mixing is nonstationary. In this case, model (2) loses its physical meaning, and neither linear techniques nor our proposed algorithm work well unless a way to introduce nonstationary seethrough is found. However, the results obtained by our procedure are interesting. In Fig. 5, we show two restored versions of the recto side: on the left, the result of the first iteration of our procedure and, on the right, the result obtained at convergence. Note that, with the initialization suggested, the left-hand image should be similar to the result obtained by the algorithm proposed in [15], which works by substituting the interfering patterns with the observed ones (*i.e.*, in terms of densities, D_r^{obs} and D_v^{obs} are used instead of D_r and D_v, respectively). From both results, it can be seen that the seethrough patterns have not been removed completely, but a significant enhancement in readability has been obtained. For example, the stamp that shows through in the recto lower-right corner has been completely removed. Note also that the spreading of ink has been significantly reduced by effect of the estimated psf. The most apparent difference between the two images is that the "final" solution shows a better contrast if compared to the "in-

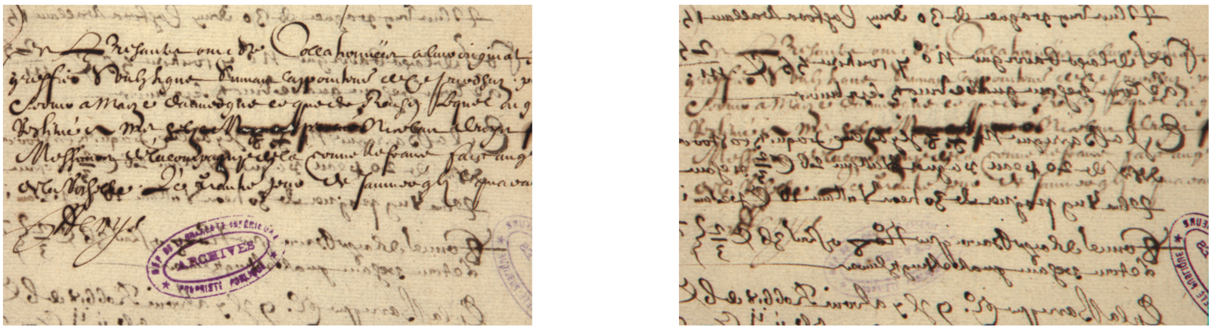

Fig. 4 Recto-verso RGB scan of a heavily distorted manuscript (after the acquisition campaign of Project Isyreadet - http://www.isyreadet.net).

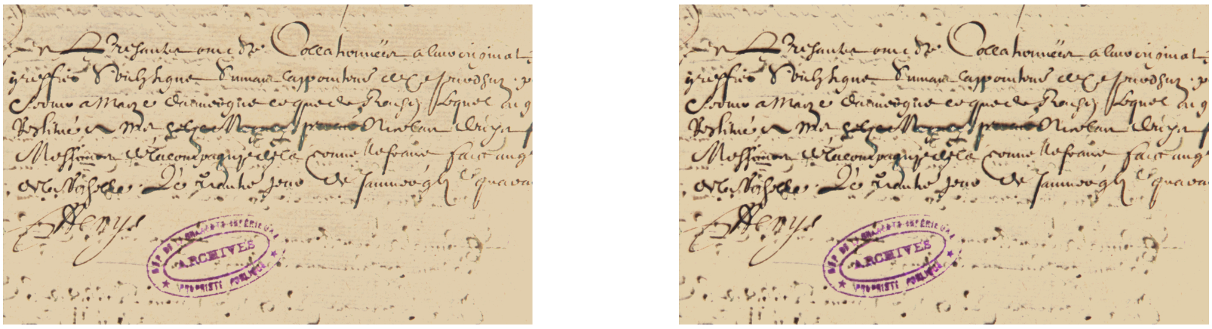

Fig. 5 Restored recto from the document in Fig. 4. Left: first iteration. Right: final result.

termediate" one. By inspecting the shapes of the restored characters, it is also possible to note that some of them are somewhat eroded with respect to the originals. This effect is progressively mitigated throughout the iteration (although typically less than ten iterations are needed to converge). In Fig. 6, we show the seethrough psf, h_{rv}, estimated for the red channel. Its peak is offset by $(+3, +4)$ pixels with respect to the center of its support. As mentioned, this corresponds to a small translational registration error, which was corrected by our procedure. Since there is no qualitative difference, we do not show the psfs estimated for the other channels, although their shapes and locations are slightly different from the ones shown in the figure. This is not surprising, since either the optical features of the capture equipment or accidental errors during acquisition normally produce different focuses and displacements in different channels even from the same document side [20].

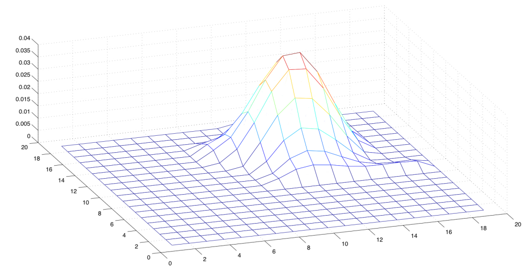

Fig. 6 Seethrough psf estimated for the red channel in the case shown in Fig. 4.

The third example comes from a grayscale two-sided scan of a printed document showing seethrough, some stains, and a rather nonuniform background, with different average levels on the recto and verso sides. The original scans and the restored images are shown in Figs. 7 and 8, respectively. The seethrough has been effectively reduced, whereas the patterns that do not originate from a recto-verso interference have not been affected. This is the case, *e.g.*, with the large stain in the verso, the handwritten strokes in the recto, and with the words "ANNALI" and "ECONOMIA", barely visible in the recto as the result of a transparency from the next page of the document.

5 Conclusion

We presented a seethrough cancellation algorithm for ancient documents based on a nonlinear image model already proposed elsewhere [15]. Rather than linearizing the model or approximating the maps of the interfering patterns, we propose to implement a procedure where the transparency of the support and the seethrough psf are estimated offline (a stationary phenomenon being assumed), and the estimates of the recto and the verso patterns are refined iteratively. From our experiments, we note that many of the inconve-

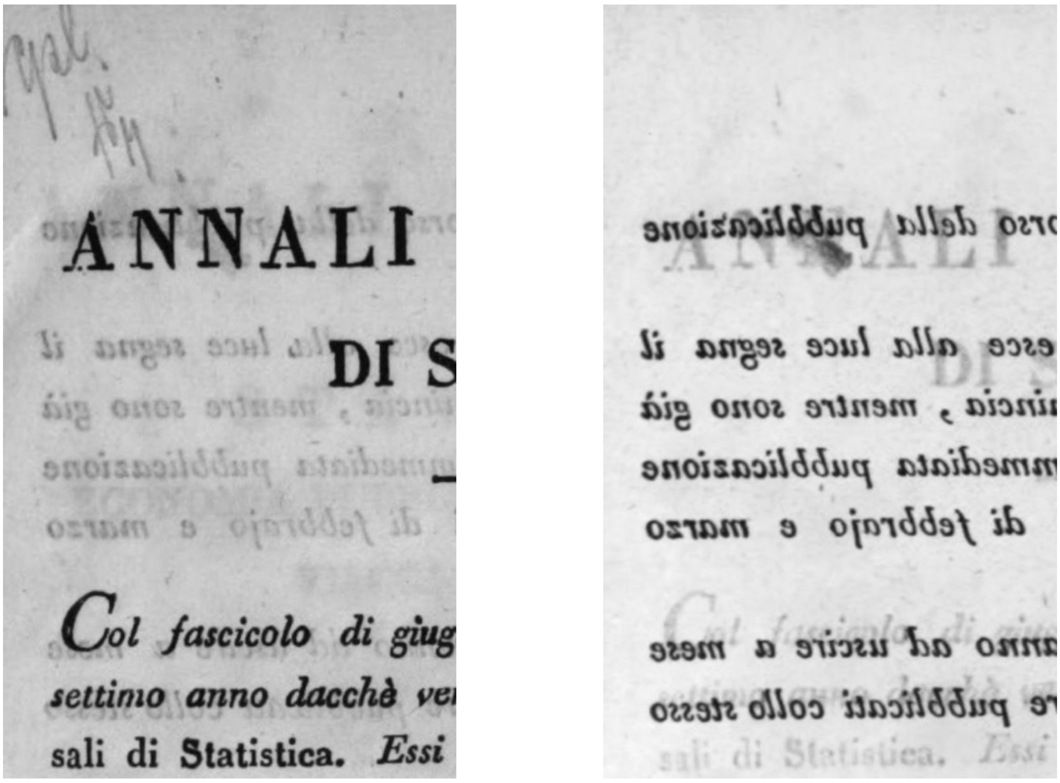

Fig. 7 Grayscale recto-verso scan of a printed document (after Google Book search dataset, 2007)

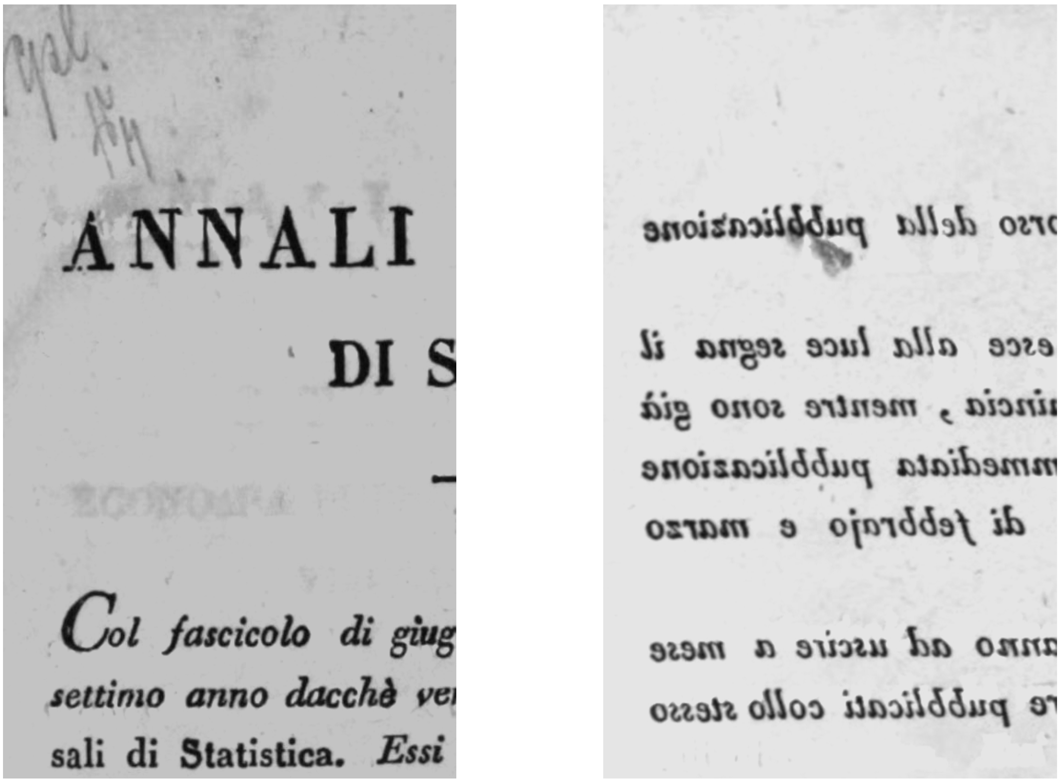

Fig. 8 Restoration results from the data in Fig. 7.

niences shown by linear and instantaneous techniques are avoided. Another advantage of the proposed method is its low computational complexity. The times required to run our procedure on a pc with RGB data images such as the ones presented here (1 to 8 Mpixels per channel) are a few tens of seconds at most.

This procedure is quite easy to implement and does not require particular skills to the operators, so it can be readily included in a document image restoration package. Its performance, however, needs to be further assessed and compared to other techniques, among which the one envisaged in Section 2, which is expected to give better results in a broader range of cases but with an increased computational cost. Also, a strategy to cope with nonstationarity is still needed. This could be accomplished, for transparency, by letting the shape of the psf be invariant and using Eq. (6) to update q at each pixel. Evaluating a space-varying offset would be more difficult by following the same strategy we present here, since it relies on an educated choice by the operator. A way to overcome this difficulty is now being studied.

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