Fundamental physics in the Field of the Earth with the LAser RAnged Satellites Experiment (LARASE)

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***Abstract*—LARASE represents a new experiment whose main goal is to provide accurate measurements for the gravitational interaction in the weak field and slow motion limit by means of the laser tracking of satellites orbiting around the Earth. Among the various ingredients needed, two of them play a very significant role: i) the quality of the tracking observations of the satellites orbit, and ii) the quality of the dynamical models implemented in a software code whose goal is to minimize, opportunely, an observable function and solve for the unknowns in which we are interested. Indeed, these models have to account for both gravitational and non-gravitational forces in such a way to reduce as better as possible the difference between the *observed* range and the *computed* (from the models) one. Of course, the better the minimization process through the orbit data reduction from one side and the better the estimate of the systematic error sources from the other, more precise and accurate will be the *a posteriori* reconstruction of the satellite orbit. Therefore, LARASE aims to improve the dynamical models of the current best laser-ranged satellites, as well as to improve the error budget estimates of the several perturbations that influence their trajectory around the Earth. This will allow to test in a reliable way Einstein's theory of general relativity with respect to other metric and non-metric theories for the gravitational interaction and to go beyond with respect to the present measurements as well as to the kind of tests carried out so far.**

I. INTRODUCTION

After 100 years from its formulation in 1915, and after a wide number of experimental verifications [1] during last 50 years, Einstein's theory of general relativity (GR) still represents the best theory for the description of the gravitational interaction [2], both at the high and low energy scales, and it is the pillar of modern cosmology to understand the universe that we observe.

However, other gravitational theories different from GR have been proposed. Some of these share with GR the same spacetime structure and the same equations of motion for test particles, but differ in the field equations form. Conversely, other theories provide more fundamental differences, such as violations of Einstein Equivalence Principle (EEP). The former are metric theories for the gravitational interaction while the latter are non-metric ones.

For instance, metric theories different from general relativity provide additional fields, beside the metric tensor $g_{\mu\nu}$ of GR, that act as "new" gravitational fields. Such additional fields may be scalar, vectorial or tensorial in their nature. The role of these fields is to "explain" how the matter and the non-gravitational fields generate the gravitational fields themselves and produce the metric. EEP is at the basis of GR and of all metric theories of gravity.

Moreover, the overall validity of Einstein's GR is not only questioned by alternative theories for the description of the gravitational interaction, but also from quantum theories of physics. Indeed, the (possible) existence of additional fields in mediating the gravitational interaction is also predicted by modern theories of physics which aim to unify gravity with the quantum realm. In fact, Einsten's GR is a classical theory of physics and all the attempts at merging gravitation with the other interactions of nature, in order to encompass all physics in a New Standard Model, have failed. Therefore, it is clear, as a consequence of the above (general) considerations, that it is very important to precisely test the consequences of GR, as well as those of competing theories, at all the accessible scales of distances and energies.

II. LARASE GOALS

It is precisely in the weak field and slow motion limit (WFSM) of GR that the LAser RAnged Satellites Experiment (LARASE) aims to provide an original contribution in testing and verifying the gravitational interaction by means of the powerful Satellite Laser Ranging (SLR) technique [3] together with a precise orbit determination (POD) of a dedicated set of passive laser-ranged satellites [4], [5].

In order to obtain a refined POD, two fundamental aspects need to be carefully reached: i) high quality for the tracking observations of the satellite, and ii) reliable dynamical models to be included in a software for the orbit determination of the satellite. Consequently, a major goal of LARASE is to improve the dynamical models of the current best laser-ranged satellites, with a special attention to the subtle, and quite complex to model, non-gravitational perturbations.

With regard to the relativistic measurements to be carried out, LARASE will mainly focus on the GR precessions related with the Earth's gravitoelectric and gravitomagnetic fields. The former field, analogous to the electric field due to electric charges of Coulomb's law, is produced by the Earth's mass, while the latter, analogous to the magnetic field due to electric currents, is produced by the Earth's current-of-mass, i.e., by the Earth's angular momentum.

These relativistic precessions are i) Einstein (or Schwarzschild) [2] and ii) Lense-Thirring (LT) [6], [7] precessions. The former precession arises from the gravitoelectric field, while the latter is due to the gravitomagnetic one. These precessions are responsible of long-term and secular effects on two of the three Euler angles that define the orbit orientation in space, namely the argument of pericenter, ω, which is subject to both precessions, and the right ascension of the ascending node, Ω, which is subject to the Lense-Thirring one.

For the secular effects we have the following (well known) expressions:

$$\dot{\omega}^{\text{Schw}} = \frac{3(GM_{\oplus})^{3/2}}{c^2 a^{5/2}(1-e^2)}, \tag{1}$$

for the gravitoelectric precession of the argument of pericenter, and

$$\dot{\omega}^{\text{LT}} = \frac{-6GJ_{\oplus}}{c^2 a^3 (1-e^2)^{3/2}} \cos i, \tag{2}$$

for its gravitomagnetic part, and

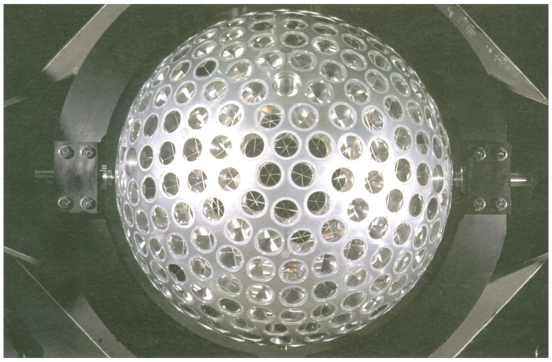
$$\dot{\Omega}^{\text{LT}} = \frac{2GJ_{\oplus}}{c^2 a^3 (1-e^2)^{3/2}}, \tag{3}$$

for the gravitomagnetic precession of the satellite node.

In the above equations, G and c are, respectively, the gravitational constant and the speed of light, $M_{\oplus}$ and $J_{\oplus}$ represent the mass and angular momentum of the Earth, finally, a, e and i, are, respectively, the orbit semi-major axis, eccentricity and inclination.

In the field of the Earth, the best measurements for the Lense-Thirring effect and for the advance of the argument of pericenter are those described respectively in [8] and in [9]. Finally, because a way to test the predictions of Einstein's GR with respect to those of other metric theories is through the measurements of the so-called parameterized post-Newtonian (PPN) parameters [10]–[13], a few of them (namely γ, β and α_1) will be the subject of the investigations and measurements of LARASE.

In the following, some of the results that we have recently obtained on the improvements of the models for some of the main perturbations acting on the two LAGEOS satellites and on LARES will be introduced.



(a) Picture of LAGEOS II (courtesy of ASI)

(b) Model of LAGEOS II at the finite elements

Fig. 1. LAGEOS II is one of the best tracked satellites all over the world by the SLR technique. LAGEOS II is almost twin of the older LAGEOS. The satellites structure is constituted by two hemispheres of aluminum containing 426 cube corner retroreflectors (CCRs), a brass core that contributes to increase the mass of the satellite and a Copper-Beryllium shaft that allows to fasten the different parts of the satellites. The heavy brass core was necessary to guarantee a low area-to-mass ratio for the two satellites, about 6.95×10^{-4} m^2/kg, the smallest among the laser-ranged satellites till the recent launch of LARES. LAGEOS II orbit has an inclination of about 53° over the Earth's equator, a semi-major axis of about 12,163 km and an eccentricity of about 0.014. LAGEOS has an orbit inclination of about 110°, a semi-major axis of about 12,270 km and an eccentricity of about 0.004. The smaller inclination of LAGEOS II has been chosen to obtain a better visibility from the network of the Earth laser ranging stations.

III. PRELIMINARY RESULTS

The test masses of the LARASE experiment will be some of the best laser-ranged satellites orbiting the Earth. These satellites are spherical in shape, fully passive, and with a generally low area/mass ratio in order to minimize the non-gravitational accelerations. In this family, the two LAGEOS and the recently launched LARES will be the most important to consider because of the high accuracy of their orbit determination thanks to the very precise SLR technique. The older LAGEOS (LAser GEOdynamic Satellite) was launched by NASA on May 4, 1976, LAGEOS II was jointly launched by NASA and ASI on October 22, 1992, finally LARES (LAser RElativity Satellite) was launched by ASI on February 13, 2012. In Figure 1, a photo of LAGEOS II and our model at the finite elements of the satellite are shown.

Fig. 2. A rendering of the LARES satellite. LARES is made of a unique piece of tungsten (THA-18N, composition 95% of W and 5% of Cu and Ni) and its surface is covered with 92 CCRs for SLR tracking. The CCRs mounting system is quite similar to that of the two LAGEOS. The satellite radius is 18.2 cm and its mass is about 386.8 kg. The area-to-mass ratio is about a factor of three smaller than that of the two LAGEOS satellites. Indeed, LARES is the densest object ever launched by the man in space. The satellite orbit is almost circular with a semi-major axis of about 7820 km (corresponding to an orbital period of about 6883 s). LARES has been lunched with the qualification flight of the new European launcher VEGA. We refer to [14], [15] for further details.

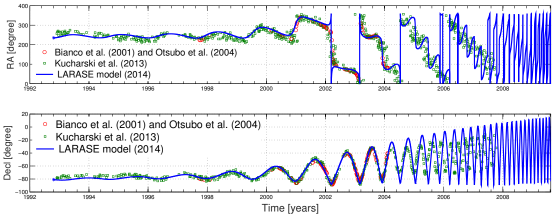
A. Internal structure

Within the activities of the LARASE program, and starting from the original drawings of the two LAGEOS satellites, we reconstructed the dimensions of the satellites and the materials used to build their internal structure. The main goal of this work was to have an independent estimate of their moments of inertia — that were not measured on the flight model of the two satellites — and to have a refined model of the satellites able to provide a reference for the development of a new thermal model in order to account properly of the quite complex perturbation produced by the thermal thrust effects. A similar work was performed also for LARES, see Figure 2.

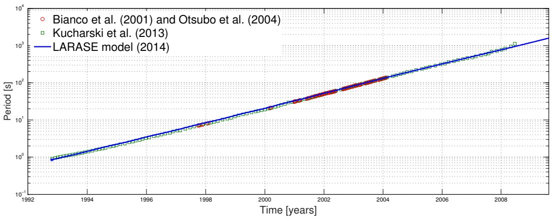
B. Spin dynamics

Indeed, with regard to the non-gravitational perturbations acting on these satellites, and in the context of the LARASE activities, we have initially focused our attention on the spin dynamics. The rotational dynamics of passive satellites like the two LAGEOS has been deeply investigated in the past by many authors (see [16]–[22]). Several non-gravitational perturbations depend on the knowledge of the satellite spin vector orientation and rate, among these, the mentioned thermal forces are the most important to consider.

In our recent work we have reviewed deeply the interactions responsible of the spin evolution of the two LAGEOS and we have removed many of the simplifications at the basis of previous models. The knowledge of the moments of inertia of a satellite plays a significant role in the case of the gravitational torque. Very important is also a reliable model of the magnetic torque, which also plays a central role. In Figure 3, in the case of the LAGEOS II satellite, the comparison between our spin model — in the so-called rapid-spin approximation — with the available observations is shown.



(a) LAGEOS II right ascension (top) and declination (bottom)



(b) LAGEOS II rotational period

Fig. 3. LAGEOS II spin orientation (a) and period (b) in the J2000 reference frame. The units are degrees for the two spherical equatorial coordinates (α, δ) and seconds in the case of the rotational period P. The results for the spin evolution, as we obtained from our model (blue line), are compared with the available observations in the literature, see [23], [24] and [25].

We have also generalized the previous model, based on the rapid-spin approximation and on averaged equations for the torques (see for instance [16]), at the more general case of non-averaged equations. This work has been also extended to LARES.

C. Neutral drag

Because of its much lower orbit with respect to that of the two LAGEOS (1450 km height vs. 5900 km), the impact of the neutral drag on the LARES orbit will be much stronger with respect to its effect on the orbit of the two LAGEOS satellites. We therefore reviewed the drag effects on the orbit of such satellites. Among the different activities that we started on this topic, we mainly focused on i) the comparison of the predictions of the different atmospheric models at the altitudes of the satellites, ii) the estimate of the perturbing accelerations acting on the satellites and iii) the estimate of the disturbing effects on their orbit. In particular, we took advantage of the use of the software SATellite Reentry Analysis Program (SATRAP) that is able to load several different models for the Earth's atmosphere together with the appropriate geomagnetic and solar activities indices, see [26], [27] for details.

Therefore, with SATRAP we have been able to investigate directly the impact of the neutral drag on the satellites orbit using the current best available models for the atmospheres main constituents. This is also the first step to be performed in order to distinguish the orbital disturbing effects of neutral drag from those of charged particle drag. In Figure 4, in the case of the Naval Research Laboratory MSISE-2000 [28] atmospheric model, we show the comparison for the values of the air density at the heights of the three satellites we are analyzing.

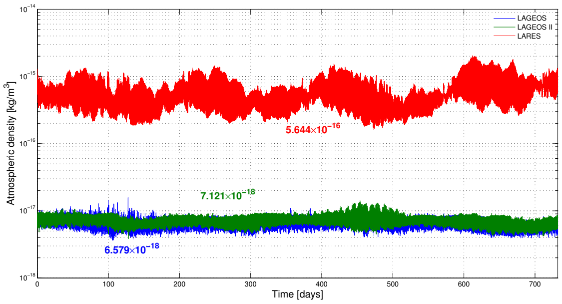


Fig. 4. Variation of the air density ρ [kg/m^3] due to the neutral drag at the altitude of the orbits of LAGEOS, LAGEOS II and LARES. The profiles have been obtained from SATRAP using NRLMSISE-2000 to model the behavior of the Earth's atmosphere. The time span covers two years since March 10, 2012. The average densities over the analyzed time span are: $6.6 \cdot 10^{-18}$ kg/m^3, $7.1 \cdot 10^{-18}$ kg/m^3 and $5.6 \cdot 10^{-16}$ kg/m^3, respectively for LAGEOS, LAGEOS II and LARES.

TABLE I

AVERAGE ACCELERATIONS [M/S^2] IN THE GAUSS REFERENCE SYSTEM FOR LAGEOS II AND LARES. IN THE CASE OF LAGEOS II, THE AVERAGE HAS BEEN COMPUTED OVER A TIME SPAN OF ABOUT 4017 DAYS, STARTING FROM JANUARY 1, 1993. IN THE CASE OF LARES, THE AVERAGE HAS BEEN COMPUTED OVER A TIME SPAN OF ABOUT 764 DAYS, STARTING FROM MARCH 10, 2012.

Acceleration component	LAGEOS II	LARES
Radial	7.5×10^{-18}	-1.3×10^{-15}
Transversal	-2.6×10^{-13}	-1.3×10^{-11}
Out-of-plane	7.1×10^{-18}	-1.8×10^{-14}

In Table I we show the results for the three components of the acceleration of the neutral drag perturbation for LAGEOS II and LARES. As we can see, despite its smaller value for the area-to-mass ratio, in the case of LARES the accelerations are much larger than those obtained for LAGEOS II. This is of course due to the higher values for the atmosphere density at the height of LARES with respect to the density "felt" by the two LAGEOS, at their much higher altitude, as well shown in Figure 4.

D. Tides

Concerning the gravitational perturbations, we have reviewed the effects provoked by both solid and ocean tides on the satellites orbit. Their effects are particularly important, in the case of relativistic measurements, on the longitude of the ascending node and on the argument of pericenter. The solid tides account for about 90% of the total response to the Moon and Sun tidal disturbing potential and are responsible of the larger effects on the orbit of a satellite. Conversely, the ocean tides are difficult to be modeled because of the greater complexity of the involved phenomena, and are characterized by larger uncertainties with respect to those of solid tides. In Table II are shown the results (amplitude and period) we obtained for a few solid (zonal) tides in the case of the ascending node of the three satellites. As we can see, in the case of the LARES satellite, due to its lower height with respect to that of the two LAGEOS, the perturbation provoked by the tides has a larger impact on the orbit.

TABLE II

IMPACT OF THE EARTH'S SOLID ZONAL TIDES ($\ell = 2$ AND $m = 0$) ON THE RIGHT ASCENSION OF THE ASCENDING NODE Ω OF THE TWO LAGEOS AND LARES. THE PERIODS ARE IN DAYS WHILE THE AMPLITUDES ARE IN MILLI ARC SECONDS. THE POSITIVE SIGN (+) OF THE PERIOD REFERS TO WESTWARD TIDAL WAVES, WHILE THE NEGATIVE SIGN (−) REFERS TO EASTWARD ONES.

Tide	Period	LAGEOS	LAGEOS II	LARES
055.565	+6798.38	-1080.22	1976.46	5332.68
055.575	3399.19	-5.23	9.57	25.81
056.554 S_a	365.25	9.97	-18.24	-49.20
057.555 S_{sa}	182.625	31.15	-56.99	-153.75

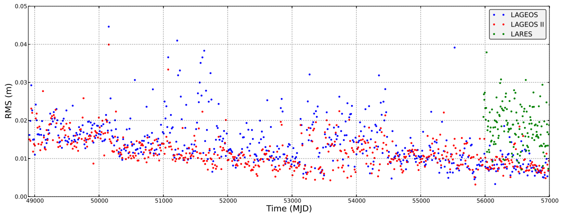


Fig. 5. Root Means Square (RMS) of the range residuals of LAGEOS (blue), LAGEOS II (red) and LARES (green), the units are in meters. The time is given in Modified Julian Date (MJD). In the case of the two LAGEOS, the starting epoch (MJD 48919) corresponds to October 24, 1992, while, in the case of LARES, the starting epoch (MJD 55975) corresponds to February 18, 2012. The arc length is 14 days for LAGEOS and LAGEOS II, and 7 days for LARES. Notice the higher uncertainty associated with the LARES analysis, showing its currently non-optimal modelling.

E. Orbit determination

As we briefly underlined in section II, a POD for the orbit of the considered satellites is an essential prerequisite in order to perform reliable measurements in gravitational physics (see [29]–[32] and [8], [9], [33]–[35]), as well as in space geodesy applications (see [36]–[42]). Our preliminary analyses included a preparatory data reduction for the satellites orbit with a tailored setup for the models implemented in the software. Together with the satellites state vector and selected station biases, the radiation coefficient and the corrections to polar motion and length of day have been estimated. Empirical acceleration have been also used. The results of these preliminary analyses are shown in Figure 5.

As we can see, LAGEOS and LAGEOS II orbits are recovered with a mean error roughly between 1.5 and 1 cm, while LARES orbit has a slightly higher error. This is due to a currently non-optimal modelling for the dynamics of LARES. The decreasing trend that we obtained for the RMS of the range residuals in the case of the two LAGEOS, which approaches the 5 mm (mean) value at the end of the time span, is also in quite good agreement with the results obtained from the data reduction of the orbit of the two LAGEOS satellites performed by the main Analysis Centers of the ILRS network.

IV. CONCLUSIONS

We have briefly introduced and described the objectives and preliminary activities of the LARASE program. Although the ultimate goal is to perform precise relativistic measurements in the weak field of the Earth through the laser measurements of the two LAGEOS satellites and of LARES, in the meantime a careful re-analysis of the existing dynamical models for such satellites, as well as the development of new and more reliable models, have been started. Such activities are important because they provide the basis to perform a POD of the orbit of the laser-ranged satellites and, at the same time, they allow for the estimate of an accurate error budget for the relativistic parameters to which we are interested in. Dedicated papers are in preparation with the details of the improvements in the modelling and the results obtained.

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