New Measurements of Gravitation in the Field of the Earth and the LARASE Experiment

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***Abstract*—We present some of the recent results of the LARASE experiment in terms of the improvements reached in the orbit determination of the LAGEOS and LARES satellites, in the development of new models for the gravitational and non-gravitational forces that produce a deviation of their orbit from that of the ideal keplerian two-body problem of Celestial Mechanics, and, finally, for relativistic measurements and tests of Einstein's theory of General Relativity. In particular, for the non-gravitational perturbations, we focus on the results of a thermal model we have developed to handle the solar Yarkovsky-Schach effect on the orbit of the older LAGEOS satellite. In this case, we also describe the model for the evolution of the spin of the satellite. The results for the model predictions of the orbit evolution are compared with the residuals in the orbit obtained by an independent analysis. Concerning the relativistic measurements, we discuss a new measurement of the precession of the orbit caused by the Lense-Thirring effect. The role of the errors related to the knowledge of the gravitational field of the Earth in this measurement is also discussed.**

I. INTRODUCTION

LARASE (LAser Ranged Satellites Experiment) [1] aims to perform new refined measurements of gravitation in the weak-field and slow-motion (WFSM) limit of Einstein's theory of General Relativity (GR) [2]. The final goal of the experiment is to test the tiny predictions of GR with respect to those of alternative theories of gravity [3]. The "test particles" of LARASE are laser-ranged satellites tracked precisely by the Satellite Laser Ranging (SLR) technique of the International Laser Ranging Service (ILRS) [4]. These are the two LAGEOS (LAser GEOdynamic Satellite) [5] and LARES (LAser RElativity Satellite) [6] satellites. In principle, after the removal of the main non-gravitational perturbations on their orbit, the satellites behave like "test particles" that fall along a geodesic in the field of the Earth and in that of the bodies of the Solar System. The improvements reached by the LARASE collaboration during the last year are presented and compared with respect to previous results [7]. In particular, the improvement reached in the orbit determination of the three satellites are described. Concerning the non-gravitational perturbations, we focus upon the modelling of thermal effects and on the spin. A new measurement of the Lense-Thirring precession [8]–[11] on the *combined orbits* of the satellites is described, and the error budget of the measurement is briefly discussed. Finally, our conclusions and recommendations on the above arguments are provided in view of further improvements.

II. ORBIT DETERMINATION

The precise orbit determination (POD) of the two LAGEOS and LARES satellites is obtained by means of the GEODYN II software of NASA/GSFC [12], [13]. Currently, we are able to fit the orbits of the two LAGEOS satellites with a root-mean-square (RMS) better than 1 cm when empirical accelerations are used to absorb some of the unmodelled thermal effects, and at a level of a few cm for LARES.

In Figures 1, 2 and 3 we show the results of the POD for the three satellites when empirical accelerations have been estimated for each arc ($\cdot$), and when empirical accelerations have not been included in the data reduction of each orbit ($+$). In these figures, the mean of the RMS of the range residuals has been plotted for each arc in which the time span of the

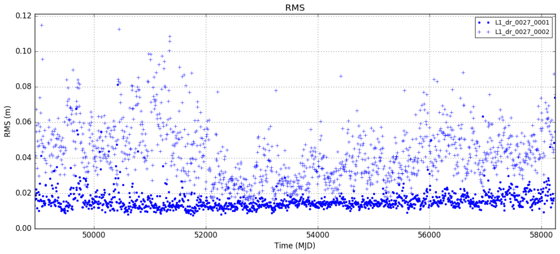

Fig. 1. Post-fit RMS of the mean range residuals of the POD of LAGEOS.

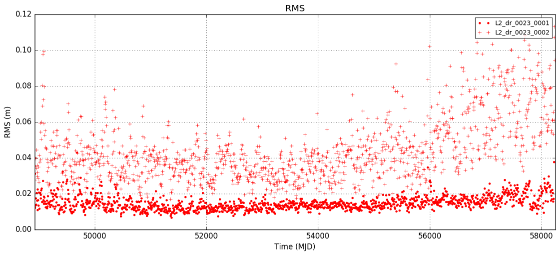

Fig. 2. Post-fit RMS of the mean range residuals of the POD of LAGEOS II.

analysis has been divided. The current results are in line with those obtained last year for the POD [7], which had been performed over a shorter time frame. In fact, the analyses cover a time span of about 25 years in the case of the two LAGEOS satellites, from October 30, 1992 (MJD 48925) to February 16, 2018 (MJD 58165), and a time span of about 5 years in the case of LARES, from April 6, 2012 (MJD 56023).

These are two standard analyses that we routinely perform. We used: i) the EIGEN-GRACE02S solution to model the gravitational field of the Earth [14]; ii) an arc length of 7 days; iii) and all tracking stations have been weighted equally. Neither the Lense-Thirring effect nor the thermal effects have been modelled. The radiation coefficient C_{R} of the satellites is usually estimated along with the empirical accelerations.

As we can see from these figures, when the empirical accelerations are not estimated, the lack of a refined modelling of the satellites orbits is quite apparent, especially in the case of LARES. However, it is important to stress that the use of empirical accelerations represents an alternative to manage those phenomena — not modelled or even unexplained — that affect the POD of the satellites. Of course, it is in the best interest of science to try to understand and to properly model these effects, rather than absorbing them in the data reduction of the orbit. Consequently, it is of crucial importance to improve the dynamical model of the orbit of the considered satellites to reduce the use of the empirical accelerations for the benefit of the measurements in the fields of gravitational physics, space geodesy and geophysics.

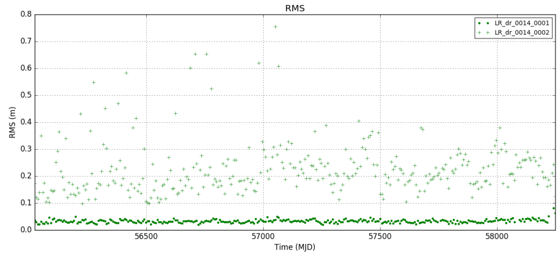

Fig. 3. Post-fit RMS of the mean range residuals of the POD of LARES.

III. THERMAL EFFECTS AND SPIN EVOLUTION

The non-uniform distribution of temperature across the surface of a satellite is responsible for the thermal thrust. Among the thermal effects, the two main perturbations to be considered are the Earth-Yarkovsky effect [15]–[17] and the solar Yarkovsky-Schach effect [15], [18]–[21].

In this paper we consider the perturbation produced by the Yarkovsky-Schach effect on the orbit of the older LAGEOS satellite. This effect is characterized by an anisotropic emission of thermal radiation from the satellite that arises from the presence of temperature gradients across its surface. These gradients are due to the solar heating and the thermal inertia of the various elements of the satellite, in particular its cube-corner retro-reflectors (CCRs).

This radiation is responsible for very subtle and complex effects, which translate into long-term perturbations on the orbital elements of a satellite. To model properly these effects are necessary a reliable thermal model able to account for the temperature distribution of the satellite, and also a very good knowledge of the spin vector of the satellite (i.e. its orientation and rate). For this reason, we developed the LASSOS spin model for the two LAGEOS and LARES satellites.

The LArase Satellites Spin mOdel Solutions (LASSOS) is based on the solution of the full set of Euler equations for the main torques acting on the considered satellites. This model has been already introduced in previous papers [22]–[24] and is fully described in [25]. Therefore, LASSOS is a general model and provides a solution for the spin evolution for any value of the satellite rotational period. In the fast-spin limit, LASSOS correctly reproduces the results of previous models [26]–[28].

In Figures 4 – 6, after a fit to the available observations [29], the predictions of LASSOS for the components of the spin orientation of LAGEOS in the J2000 reference frame and for its rotational period are shown. The analysis covers a time span of about 34 years, starting from May 15, 1976 (MJD 42913). As we can see, the agreement between the LASSOS model for the evolution of the spin of LAGEOS and the available observations is quite good.

In order to build a reliable model for the various thermal effects, we need a deep knowledge of the physical characteristics of the various elements that constitute the satellite's surface and interior. We have solved the problem in two different ways: we first developed a simplified thermal model based on averaged equations, as in [18], [20], [30], [31] then we developed a general thermal model not restricted to averaged

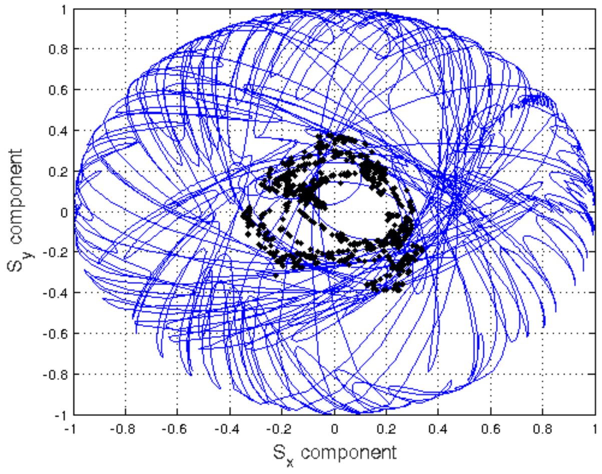

Fig. 4. Prediction of LASSOS for the spin of LAGEOS over a time span of 12487 days. Projection of the satellite spin direction on the equatorial plane (S_x, S_y) compared with observations (black dots) as obtained in [29].

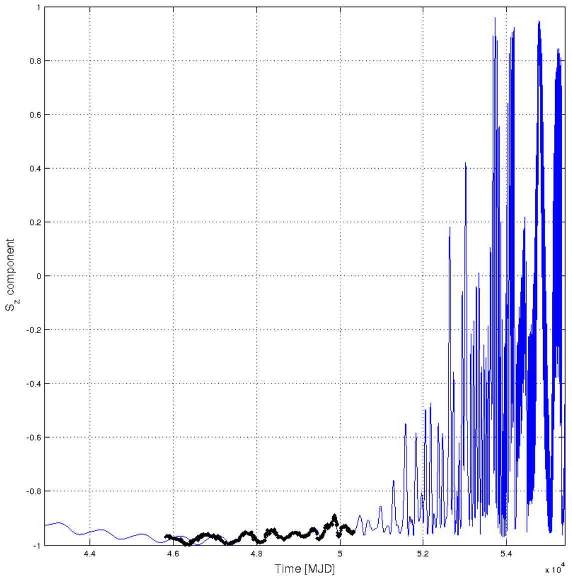

Fig. 5. Prediction of LASSOS for the spin of LAGEOS over a time span of 12487 days. Component of the satellite spin direction along the vertical axis (S_z) of the J2000 reference frame compared with observations (black dots) as obtained in [29].

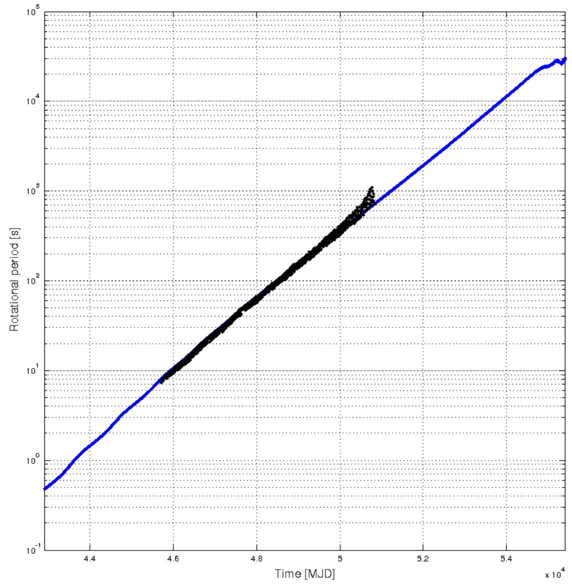

Fig. 6. Prediction of LASSOS for the spin of LAGEOS over a time span of 12487 day. Rotational period of the satellite [s] compared with observations (black dots) as obtained in [29].

equations, as in [21], [32]. The first approach is simpler and more practical in the fast spin approximation.

In this simplified case, the thermal model is mainly based on the application of the energy balance equation on the satellite surface, and on a linear approximation for the distribution of the temperature of the CCRs with respect to the satellite equilibrium temperature. We present the results for LAGEOS in this case.

In Figure 7, the components of the acceleration due to the Yarkovsky-Schach effect in the Gauss co-moving frame of the satellite are shown. The analysis covers the same time span of the one performed for the spin evolution of the satellite.

The green line represents the radial component of the acceleration, along the Earth-to-satellite direction. The red line represents the normal (or out-of-plane) component of the acceleration, normal to the orbital plane, along the direction of the osculating angular momentum. Finally, the blue line represents the transversal component of the acceleration, orthogonal to the other two components.

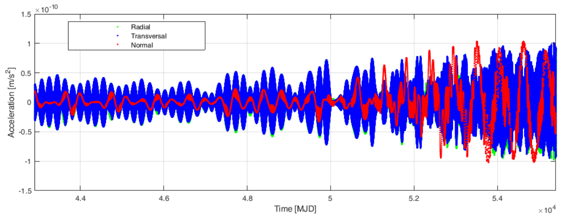

Fig. 7. Gauss perturbing accelerations on LAGEOS due to the Yarkovsky-Schach effect over a time span of about 34 years. These results have been obtained by the simplified thermal model developed by the LARASE experiment.

In Figures 8 and 9, the perturbing effects on the satellite eccentricity and argument of pericenter are shown and compared with the residuals in these elements obtained by our POD.

The comparison between the model and the residuals in the same elements has been performed over a time span of 9 years starting from November 4, 1992 (MJD 48930). In this analysis, the arc length was 7 days, no empirical

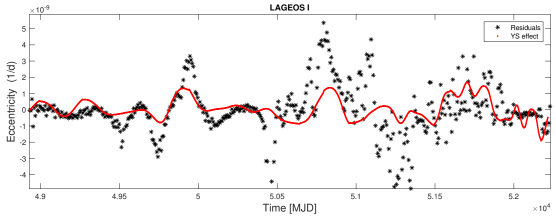

Fig. 8. Yarkovsky-Schach effect on LAGEOS eccentricity. The predictions of the simplified thermal model (red line) are compared with the residuals in the eccentricity (black line) obtained by means of a POD performed over the same time span of 4080 days.

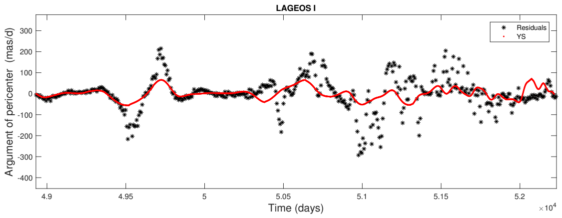

Fig. 9. Yarkovsky-Schach effect on LAGEOS argument of pericenter. The predictions of the simplified thermal model (red line) are compared with the residuals in the argument of pericenter (black line) obtained by means of a POD performed over the same time span of 4080 days.

accelerations were used and the thermal effects (Yarkovsky-Schach and Earth-Yarkovsky) were not modelled. The EIGEN-GRACE02S solution has been used as model of the Earth gravitational field [14].

As we can see from these two figures, the agreement between the prediction of the model and the residuals is good, but not perfect. However, the agreement could not be perfect since, besides the signature of the Yarkovsky-Schach effect, these residuals also contain the signature of the Earth-Yarkovsky effect as well as that of the asymmetric reflectivity of the satellite's hemispheres [33], [34].

Thermal perturbations share with tides some frequencies that perturb the main observables used by LARASE for the measurement of the relativistic effects, as the right ascension of the ascending node Ω and the argument of pericenter ω. Therefore, it is of the utmost importance to model them in a very reliable manner, otherwise their long-term effects may corrupt the measurement of the relativistic precessions.

IV. A NEW MEASUREMENT OF THE LENSE-THIRRING EFFECT

In the following, we present a new precise measurement of the relativistic precession of the orbit of a satellite due to the Lense-Thirring effect of Einstein's GR. This precession is produced by the Earth's angular momentum and it is effective on the right ascension of the ascending node and on the argument of pericenter, i.e. on two of the three angles that define the orientation of the orbit in space.

To be more precise, the measurement is based on the combined analysis of the orbits of the two LAGEOS and LARES satellites, and for the relativistic observable, the right ascension of the ascending node has been used, since it is less perturbed than the argument of pericenter by the thermal effects.

We performed a POD of the orbit of the satellites over a time span of about 4.7 years, starting from April 6, 2012 (MJD 56023). For the arc length we used 7 days, no empirical accelerations have been used and the thermal effects have not been modelled. For the background gravitational field of the Earth, the GGM05S model [14], [35], [36] has been used.

This new measurement represents an improvement of the preliminary measurement that we worked out in 2017 and presented in [7]. In particular, the main improvement arises from a new study related with the impact on the relativistic precession of the errors due to the first even zonal harmonics of the Earth gravitational field.

In fact, from a series of dedicated analyses aimed at determining the influence of the knowledge of the low-degree coefficients on the measurement, we found that the nominal (constant) value of the third even zonal harmonic $\bar{C}_{60}$ of GGM05S is far from its independent monthly solutions provided by the Center for Space Research (CSR) of the University of Texas at Austin. These solutions show an important (and complex) time dependency [36], [37] for this coefficient, and our independent estimates of this coefficient are in good agreement with those obtained by the CSR. Consequently, if we do not take into account this behaviour for $\bar{C}_{60}$, its nominal value will be responsible of a large systematic effect on the measurement of the Lense-Thirring secular precession, at least on the time span of our analysis of the orbits of the satellites.

We remind that the reference epoch of GGM05S is January 1, 2008 (MJD 54466), and that the field was estimated on a time interval of about ten years from March 2003 through May 2013, but with several gaps and not all months being characterized by an equal number of measurements from the two GRACE satellites (John Ries, private communication).

Therefore, in our new analysis we assumed as unknowns, in addition to the general relativistic precession, the first and third even zonal harmonics, i.e. their variations $\delta\bar{C}_{20}$ and $\delta\bar{C}_{60}$ with respect to their nominal values which were used in the POD of each satellite. Conversely, in our previous preliminary measurement described in [7], as well as in all previous measurements of the Lense-Thirring effect reported in the literature, the third unkown was the second even zonal harmonic, i.e. $\delta\bar{C}_{40}$.

In the case of the $\bar{C}_{60}$ even zonal harmonic, the nominal value included in the POD was that given by the GGM05S solution, that is $-1.499751867421 \times 10^{-7}$. On the other hand, in the case of the first even zonal harmonic $\bar{C}_{20}$, the coefficient was modelled by means of a linear dependency in time, obtained by a linear fit we performed to the monthly solutions for this coefficient that have been estimated by the CSR [37].

Following the above arguments, the combined relativistic precession for the three satellites is:

$$\dot{\Omega}^{\rm LT}_{\rm comb} = \dot{\Omega}^{\rm LT}_{\rm LI} + \xi_1 \dot{\Omega}^{\rm LT}_{\rm LII} + \xi_2 \dot{\Omega}^{\rm LT}_{\rm LR}, \tag{1}$$

where the coefficients ξ_1 and ξ_2 are obtained from the solution of a linear system of three equations in three unknowns in such a way to remove the errors from the first and third even zonal harmonics, $\delta\bar{C}_{20}$ and $\delta\bar{C}_{60}$, while solving for the relativistic precession.

The overall relativistic precession $\dot{\Omega}^{\mathrm{LT}}_{\mathrm{comb}}$ is equal to 49.658 mas/yr, and can be computed, once determined the above coefficients, from the values of the relativistic precession on the right ascension of the ascending node of each satellite, as shown in Table I.

TABLE I
RATE [MAS/YR] FOR THE LENSE-THIRRING PRECESSION ON THE RIGHT ASCENSION OF THE ASCENDING NODE OF LAGEOS, LAGEOS II AND LARES.

Orbital element	LAGEOS	LAGEOS II	LARES
$\dot{\Omega}^{\mathrm{LT}}$	30.67	31.50	118.48

The result for the relativistic precession on the combined orbits of LAGEOS, LAGEOS II and LARES is shown in Figure 10.

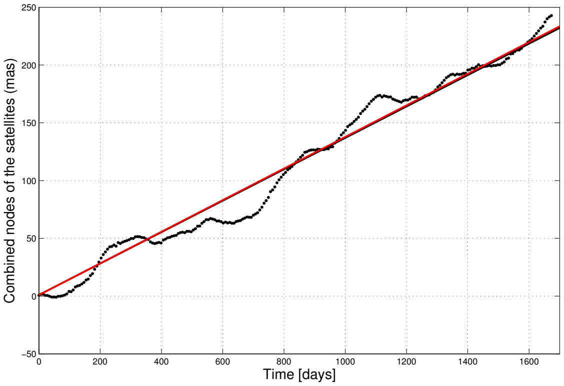

Fig. 10. Combined residuals (black dots) of the right ascension of the ascending node of LAGEOS, LAGEOS II and LARES. The continuous red line represents a linear fit to the combined residuals. The continuous black line represents the prediction of GR for the Lense-Thirring secular precession. The slope of the red line corresponds to the relativistic precession being measured, and it is practically indistinguishable from the slope of the black line.

The combined (and integrated) residuals [38] for the right ascension of the ascending node of the three satellites are shown over the time span of the analysis (black dots). The continuous black line represents the GR prediction for the combined precession of the nodes. Conversely, the red line represents the result of a linear fit to the residuals.

For the measured precession $\dot{\Omega}^{\mathrm{meas}}_{\mathrm{comb}}$ — which corresponds to the slope of the red line — we obtained a value of 49.966 mas/yr, very close to the relativistic prediction of GR. Indeed, the fractional discrepancy with respect to the GR value is very small:

$$\frac{\dot{\Omega}^{\mathrm{meas}}_{\mathrm{comb}} - \dot{\Omega}^{\mathrm{LT}}_{\mathrm{comb}}}{\dot{\Omega}^{\mathrm{LT}}_{\mathrm{comb}}} \simeq 6.2 \times 10^{-3}. \tag{2}$$

This result represents, to our knowledge, the most precise measurement obtained so far for the secular Lense-Thirring precession by means of a linear fit to the combined residuals of the orbits of the two LAGEOS and LARES satellites.

As Figure 10 clearly shows, the combined residuals of the satellites nodes are characterized by the presence of unmodelled periodic effects. As shown in [7], see their Table VI, the spectral analysis of the residuals in the right ascension of the ascending node of the three satellites relates these periodic effects to those of the unmodelled thermal forces and to some tides. In this context, the success of a simple linear fit, as the one we have done here, is related to the number of full cycles, of the unmodelled periodic effects, contained in the time interval covered by the measurement.

The analysis of the systematic effects is ongoing. Our final goal is to provide a reliable and robust error budget for the measurement of the Lense-Thirring effect. On the basis of our preliminary estimates of the various sources of error, both gravitational and non-gravitational, we can fix the error budget for the measurement of the Lense-Thirring effect at a few % level for the combination of the satellites orbits that we currently considered.

V. CONCLUSIONS

We have briefly introduced and described some of the activities performed by LARASE during the last year. The ultimate goal of LARASE [1] is to provide measurements of the gravitational interaction in the WFSM limit of Einstein's GR and to compare them with the predictions of alternative theories of gravitation [3]. This allows to place strong constraints on some of these theories, both metric and non-metric, and to further validate, or falsify, GR [39]–[41].

We discussed the improvements obtained in the POD of the two LAGEOS and LARES satellites, the results concerning the modelling of the Yarkovsky-Schach effect and a new measurement of the secular precession of the satellites orbit due to mass-currents [42], the so-called Lense-Thirring effect.

Improvements in the modelling of the various perturbations to the orbit of a LAGEOS-like satellite are an essential prerequisite to finally provide precise and accurate measurements of the relativistic effects in the field of the Earth.

For these reasons, the activities of LARASE are strongly devoted to develop new models for the non-conservative forces and to analyse carefully the models developed for the static and dynamic parts of the Earth's gravitational field, as well as of the tides [22].

Unmodelled periodic effects with a long periodicity are dangerous in this kind of measurements because they may resemble a secular-like effect over a shorter time span. These are the thermal effects and tides, especially the ocean tides. In particular, the most critical are the unmodelled long-term effects that have the period of the right ascension of the ascending node of the considered satellites, since this element represents our main observable for the relativistic measurement.

Concerning the Earth's gravitational field, we have briefly discussed the errors related with the low-degree coefficients, in particular those connected with the even zonal harmonics.

We have shown that the third even zonal harmonic of the gravitational field of the Earth is characterized by a more complex temporal pattern with respect to its nominal value in GGM05S. Therefore, since we use three observables (the nodes of the three satellites) for the measurement, we decided to estimates this coefficient together with the relativistic Lense-Thirring precession and the first even zonal harmonic.

Finally, we have described a precise measurement of the Lense-Thirring precession, obtained through a simple linear fit to the satellites residuals estimated from three PODs. The discrepancy between our new measurement and the GR prediction is just 0.6%. We plan to repeat this measurement over a longer time span and by means of a non-linear fit to absorb the unmodelled thermal thrust effects. This will provide, in the centenary of the publications by Lense and Thirring [8]–[10], a very precise and accurate measure for the relativistic precession that they, before anyone else, have highlighted.

ACKNOWLEDGMENT

The authors acknowledge the ILRS for providing high quality laser ranging data of the two LAGEOS satellites and of LARES. This work has been in part supported by the Commissione Scientifica Nazionale II on astroparticle physics experiments of the Istituto Nazionale di Fisica Nucleare.

REFERENCES

[1] D. Lucchesi, L. Anselmo, M. Bassan, C. Pardini, R. Peron, G. Pucacco, and M. Visco, "Testing the gravitational interaction in the field of the Earth via satellite laser ranging and the Laser Ranged Satellites Experiment (LARASE)," *Class. Quantum Grav.*, vol. 32, p. 155012, 2015.

[2] A. Einstein, "Die Grundlage der allgemeinen Relativitätstheorie," *Annalen der Physik*, vol. 354, pp. 769–822, 1916.

[3] C. M. Will, *Theory and Experiment in Gravitational Physics*. Cambridge, UK: Cambridge University Press, Mar. 1993.

[4] M. R. Pearlman, J. J. Degnan, and J. M. Bosworth, "The International Laser Ranging Service," *Adv. Space Res.*, vol. 30, pp. 135–143, 2002.

[5] S. C. Cohen and D. E. Smith, "Lageos scientific results - Introduction," *J. Geophys. Res.*, vol. 90, pp. 9217–9220, Sep. 1985.

[6] A. Paolozzi and I. Ciufolini, "LARES successfully launched in orbit: Satellite and mission description," *Acta Astronautica*, vol. 91, pp. 313–321, Oct. 2013.

[7] D. Lucchesi, C. Magnafico, R. Peron, M. Visco, L. Anselmo, C. Pardini, M. Bassan, G. Pucacco, and R. Stanga, "The LARASE research program. State of the art on Modelling and Measurements of General Relativity effects in the field of the Earth: a preliminary measurement of the Lense-Thirring effect," *Metrology for Aerospace (MetroAeroSpace). IEEE Xplore*.

[8] J. Lense and H. Thirring, *Phys. Z.*, vol. 19, p. 156, 1918.

[9] H. Thirring, "Über die formale Analogie zwischen den elektromagnetischen Grundgleichungen und den Einsteinschen Gravitationsgleichungen erster Näherung," *Physikalische Zeitschrift*, vol. 19, pp. 204–205, 1918.

[10] ——, "Über die Wirkung rotierender ferner Massen in der Einsteinschen Gravitationstheorie," *Physikalische Zeitschrift*, vol. 19, pp. 33–39, 1918.

[11] B. Mashhoon, F. W. Hehl, and D. S. Theiss, "On the gravitational effects of rotating masses - The Thirring-Lense Papers," *Gen. Rel. Grav.*, vol. 16, pp. 711–750, 1984.

[12] B. Putney, R. Kolenkiewicz, D. Smith, P. Dunn, and M. H. Torrence, "Precision orbit determination at the NASA Goddard Space Flight Center," *Adv. Space Res.*, vol. 10, pp. 197–203, 1990.

[13] D. E. Pavlis and et al., *GEODYN II Operations Manual*, NASA GSFC, 1998.

[14] C. Reigber, R. Schmidt, F. Flechtner, R. König, U. Meyer, K.-H. Neumayer, P. Schwintzer, and S. Y. Zhu, "An Earth gravity field model complete to degree and order 150 from GRACE: EIGEN-GRACE02S," *J. Geodyn.*, vol. 39, pp. 1–10, Jan. 2005.

[15] D. Rubincam, "On the secular decrease in the semimajor axis of lageos's orbit," *Celestial mechanics*, vol. 26, no. 4, pp. 361–382, 1982. [Online]. Available: http://dx.doi.org/10.1007/BF01230417

[16] D. P. Rubincam, "LAGEOS orbit decay due to infrared radiation from earth," *J .Geophys. Res.*, vol. 92, pp. 1287–1294, Feb. 1987.

[17] ——, "Drag on the Lageos satellite," *J. Geophys. Res.*, vol. 95, pp. 4881–4886, Apr. 1990.

[18] P. Farinella, A. M. Nobili, F. Barlier, and F. Mignard, "Effects of thermal thrust on the node and inclination of LAGEOS," *Astron. Astrophys.*, vol. 234, pp. 546–554, Aug. 1990.

[19] R. Scharroo, K. F. Wakker, B. A. C. Ambrosius, and R. Noomen, "On the along-track acceleration of the Lageos satellite," *J. Geophys. Res.*, vol. 96, pp. 729–740, Jan. 1991.

[20] P. Farinella and D. Vokrouhlický, "Thermal force effects on slowly rotating, spherical artificial satellites-I. Solar heating," *Plan. Space Sci.*, vol. 44, pp. 1551–1561, Dec. 1996.

[21] V. J. Slabinski, "A Numerical Solution for Lageos Thermal Thrust: The Rapid-Spin Case," *Celest. Mech. Dyn. Astron.*, vol. 66, pp. 131–179, Jun. 1996.

[22] D. M. Lucchesi, R. Peron, M. Visco, L. Anselmo, C. Pardini, M. Bassan, and G. Pucacco, "Fundamental physics in the field of the Earth with the laser ranged satellites experiment (LARASE)," in *Metrology for Aerospace (MetroAeroSpace), 2015 IEEE*, June 2015, pp. 71–76.

[23] D. M. Lucchesi, C. Magnafico, R. Peron, M. Visco, L. Anselmo, C. Pardini, M. Bassan, G. Pucacco, and R. Stanga, "Measurements of general relativity precessions in the field of the earth with laser-ranged satellites and the larase program," in *2016 IEEE Metrology for Aerospace (MetroAeroSpace)*, June 2016, pp. 522–529.

[24] D. M. Lucchesi, C. Magnafico, R. Peron, M. Visco, L. Anselmo, C. Pardini, M. Bassan, G. Pucacco, and R. Stanga, "The LARASE research program. State of the art on Modelling and Measurements of General Relativity effects in the field of the Earth: a preliminary measurement of the Lense-Thirring effect," in *2017 IEEE International Workshop on Metrology for AeroSpace (MetroAeroSpace) , pp. 131-145, 2017*, Aug. 2017, pp. 131–145.

[25] M. Visco and D. M. Lucchesi, "The LArase Satellites Spin mOdel Solutions (LASSOS): a comprehensive model for the spin evolution of the LAGEOS and LARES satellites," *ArXiv e-prints*, Jan. 2018.

[26] B. Bertotti and L. Iess, "The rotation of Lageos," *J. Geophys. Res.*, vol. 96, pp. 2431–2440, Feb. 1991.

[27] P. Farinella, D. Vokrouhlicky, and F. Barlier, "The rotation of LAGEOS and its long-term semimajor axis decay: A self-consistent solution," *J. Geophys. Res.*, vol. 101, pp. 17 861–17 872, Aug. 1996.

[28] J. I. Andrés, R. Noomen, G. Bianco, D. G. Currie, and T. Otsubo, "Spin axis behavior of the LAGEOS satellites," *Journal of Geophysical Research (Solid Earth)*, vol. 109, no. B18, p. 2994, Jun. 2004.

[29] D. Kucharski, H.-C. Lim, G. Kirchner, and J.-Y. Hwang, "Spin parameters of LAGEOS-1 and LAGEOS-2 spectrally determined from Satellite Laser Ranging data," *Adv.Space Res.*, vol. 52, pp. 1332–1338, Oct. 2013.

[30] D. P. Rubincam, "Yarkovsky thermal drag on LAGEOS," *J. Geophys. Res.*, vol. 93, pp. 13 805–13 810, Nov. 1988.

[31] G. Afonso, F. Barlier, F. Mignard, M. Carpino, and P. Farinella, "Orbital effects of LAGEOS seasons and eclipses," *Ann. Geophysicae*, vol. 7, pp. 501–514, Oct. 1989.

[32] J. I. Andrés de la Fuente, "Enhanced Modelling of LAGEOS Non-Gravitational Perturbations," Ph.D. dissertation, Delft University Press, Sieca Repro, Turbineweg 20, 2627 BP Delft, The Netherlands, 2007.

[33] D. M. Lucchesi, "The asymmetric reflectivity effect on the LAGEOS satellites and the germanium retroreflectors," *Geophys. Res. Lett.*, vol. 30, p. 1957, Sep. 2003.

[34] ——, "LAGEOS Satellites Germanium Cube-Corner-Retroreflectors and the Asymmetric Reflectivity Effect," *Celest. Mech. Dyn. Astron.*, vol. 88, pp. 269–291, Mar. 2004.

[35] B. D. Tapley, F. Flechtner, S. V. Bettadpur, and M. M. Watkins, "The status and future prospect for GRACE after the first decade," *Eos Trans. Fall Meet. Suppl. Abstract G22A-01*, 2013.

[36] M. Cheng, B. D. Tapley, and J. C. Ries, "Deceleration in the Earth's oblateness," *Journal of Geophysical Research: Solid Earth*, vol. 118, no. 2, pp. 740–747, 2013. [Online]. Available: http://dx.doi.org/10.1002/jgrb.50058

[37] M. Cheng and J. C. Ries, “Decadal variation in Earth’s oblateness (J_2) from satellite laser ranging data,” *Geophysical Journal International*, vol. 212, pp. 1218–1224, Feb. 2018.

[38] D. M. Lucchesi and G. Balmino, “The LAGEOS satellites orbital residuals determination and the Lense Thirring effect measurement,” *Plan. Space Sci.*, vol. 54, pp. 581–593, 2006.

[39] D. M. Lucchesi and R. Peron, “Accurate Measurement in the Field of the Earth of the General-Relativistic Precession of the LAGEOS II Pericenter and New Constraints on Non-Newtonian Gravity,” *Phys. Rev. Lett.*, vol. 105, no. 23, p. 231103, Dec. 2010.

[40] C. M. Will, “The Confrontation between General Relativity and Experiment,” *Living Rev. Relativity*, vol. 17, p. 4, Jun. 2014.

[41] D. M. Lucchesi and R. Peron, “LAGEOS II pericenter general relativistic precession (1993-2005): Error budget and constraints in gravitational physics,” *Phys. Rev. D*, vol. 89, no. 8, p. 082002, Apr. 2014.

[42] I. Ciufolini and J. A. Wheeler, *Gravitation and inertia*. Princeton: Princeton University Press, 1995.