Analysis of violin combination tones and their contribution to Tartinis third tone

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Abstract— It is widely accepted that the famous Tartini's third tone, i.e., the appearance of an additional third tone of lower frequency when playing a dyad on the violin, is a subjective phenomenon generated by the listener's cochlear nonlinearity. However, the recent demonstration that additional tones of audible amplitude can also be generated by the violin itself during playing of a dyad (violin combination tones), raises the question if these tones might have influenced Tartini's third sound perception. The experiments reported here were made to ascertain this possibility. To this end, following Tartini experiments, several dyads played by either one violin or two violins playing one note of the dyad each, were recorded. The analysis of the spectra shows that violin combination tones are present in all the dyads investigated, but exclusively when the dyad is played by a single violin and not when the same dyad is played by two violins. Tartini found the third tones to be the same in both conditions, which means that violin combination tones in his experiments were either absent or too small to affect the perception of the subjective third tones arising from cochlear distortion.

I. Introduction

In 1714 in Ancona (Italy) the famous violinist, composer and musical theorist Giuseppe Tartini discovered that when playing two simultaneous notes (a dyad) on the violin he could clearly hear a third tone called by him *terzo suono* [1]. This tone was weaker than the main notes and it was lower than the lower note of the dyad. The origin, characteristics and musical applications of the third tone have been thoroughly investigated since the eighteenth century. These extensive studies showed the existence of several tones, rather than one, and the term Combination Tones was introduced to describe the complexity of the phenomenon better [2].
A great step forward in understanding the nature of the combination tones was made by Helmholtz (pp. 411-413 of [3]) with the formal demonstration that a non-linear response of the auditory system, in presence of two different notes of frequency f_1 and f_2 , can lead to the formation of additional tones. These occur mainly at frequencies $f_2 - f_1$ and $f_1 + f_2$ ($f_2 > f_1$) and constitute most of the combination tones normally heard but not present in the air (distortion theory). Helmholtz called them, *subjective combination tones* to distinguish them from the *objective combination tones* generated by some musical instrument where the two sound sources are mechanically coupled as occurs in the Harmonium [3]. In contrast to subjective tones, objective tones are present and detectable in the air. More recent studies with sinusoidal tones, confirmed the substantial correctness of Helmholtz's hypothesis, and revealed the presence of additional subjective tones mainly $2f_1 - f_2$ and $3f_1 - 2f_2$ (see [4], [5]).
The auditory non-linearity was located in the amplification mechanism of the cochlea (for a review, see [6]). Recently it has been shown that objective combination tones are also produced by some bowed instruments, especially by the violin [7], [8]. Listening experiments, suggested that these combination tones, hereinafter called Violin Combination Tones (VCTs), are perceivable [7]. Hence we can assume that they add to the subjective tones to produce the tone ultimately perceived by the listener. Thus, we may ask if combination tones produced by the violin, could have had a significant impact on the perception of the third tones described by Tartini.
In fact, Tartini describes two experimental settings for studying third tones, which can be considered quite different from the physical point of view. The first one involves a violin playing the dyad, whereas in the second one, two violins five or six steps away, play at the same time one note of the dyad each. In the first setting the two sound sources are strongly coupled similarly to the Helmholtz harmonium, VCTs are therefore expected. In the second settings, the only coupling possible between the two violins occurs through the mechanism of the sympathetic resonance. Given the distance, the coupling between the two violins is expected to be very small or absent and the same is true for

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VCTs. Tartini found the third tones to be the same with both settings, which means that either the VCTs were the same in both cases or they were absent or too small to influence the subjective tones due to the cochlear distortion. It is clear that investigation of the VCTs under both conditions become essential to establish the possible impact of VCTs on the third tones perceived by Tartini. To this end we investigated the properties of Violin Combination Tones with experiments reproducing as much as possible those described by Tartini.

II. Methods and Experimental setting

In his treatise [1], Tartini states that the third tone is heard equally well either when a player plays the dyad on a single violin or when two players, five or six steps away from each other, play one note of the dyad each with the listener placed midway between them. To reproduce both these conditions, the Violin Combination Tones (VCTs) were investigated with the following protocol: two professional violinists, players 1 and 2, playing two contemporary Italian violins, were asked to play *forte* and without vibrato several intervals among those shown by Tartini (pp. 14-17 of [1]), standing at about 2.70m apart (five or six steps), following a precise execution program. Two microphones (mod.CM3, Line Audio Design, Sweden) were placed near the ears of each player (mics $1, 2$ and $3, 4$, respectively) at about 40 cm from the violin. To simulate the experience of the central listener, two further mics (mod. SE 4, SE Electronics, USA) were placed in the middle between the two players in opposition to each other looking towards the players (mics 5 and 6). Finally, one mic (mod. KM 184, Neumann, Germany) was placed in front of each player (mics 7 and 8) at about 50 cm from the violin. Signals were fed to a low-noise, flat-response preamplifier (Line Audio 8MP, Sweden) and from there to a sound card (Motu Traveler mk3, Motu, USA). Signals were recorded individually at 24bit and 192kHz with the Logic Pro software (Apple, USA) without any compression.

Processing of the audio signals and the Fourier Transforms, calculated with the Fast Fourier Transform algorithm (FFT), were performed on a period of 2s selected from recording periods of about 5s each. The selection was made by choosing for the most stable period of the note recorded. Analysis, data elaboration and statistics were made with Sigview (USA) and OriginPro software (USA). The spectra are expressed in dB relative to the greatest peak. All the plotted FFT records were filtered with the moving average method (5 points). The recording protocol, applied to all the dyads analyzed, was split into sections of about 5s each. Each player played alone and separately: (1) the dyad, (2) the note 1 (whose fundamental is f_1), (3) the note 2 (f_2) of the dyad. Further (4), player 1 and 2 played simultaneously one note of the dyad each, then (5) switched the note played. Each section was repeated three or more times. By selecting appropriately the microphone recordings, we obtained the following conditions for VCTs examination: 1) dyads performed separately by player 1 and by player 2, mics $1, 2$ and $3, 4$, both termed SPD; 2) dyads performed by the two players playing simultaneously one note of the dyad each, mics 5 and 6, termed Two Players Dyad (TPD); 3) dyads obtained by electronically adding the separated records of note 1 and note 2 of player 1, mic 7, and the same for player 2, mic 8, both termed SMD.

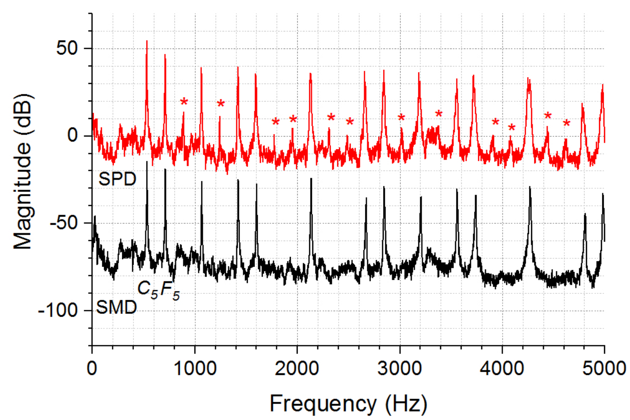

Figure 1 - FFT spectra of the interval of perfect fourth $C_5 - F_5$. Summed Notes Dyad (SMD) trace, summed note dyad, Single Player Dyad (SPD) trace, single player dyad. Peaks marked by the asterisks in SPD, are the VCTs. Note that both traces have a very similar background, fundamental and partial amplitudes, the only difference being the VCTs. Trace SPD, originally superimposed to SMD, was shifted upward and the abscissa was limited to 5kHz, for clarity.

Conditions SPD and TPD emulate Tartini's experimental settings. Condition TPD was also used by Tartini to investigate the third tone produced by the oboe, which he found to be even stronger that in violin [1]. Clearly, SMD was not possible at Tartini's time, because it requires electronic recording techniques.

Condition SPD is characterized by the strongest mechanical and acoustic coupling between the two played notes, similarly to the Harmonium. The primary sound sources (the strings) are acoustically and mechanically coupled mainly through the bridge and the violin's top plate (secondary sources) and the bow. Combination tones are therefore expected. Mechanical interaction between the two sound sources when the dyad is played by two players 2.7m apart (TPD) is clearly absent. In principle, however, some acoustic interaction cannot be excluded: in fact, air vibration induced by the note played by one violin might put into vibration the top plate of the other violin (sympathetic resonance) giving rise to some interaction between the two notes which in turn might induce VCTs generation. Due to the weak interaction between the two violins, it seems obvious to predict that VCTs, if present, are very weak, however to our knowledge this point has not been verified experimentally. The importance of the conditions in which the dyad is obtained by two notes recorded individually and summed electronically later (SMD), is that no acoustic and no mechanical interaction at all can be present. Thus the FFT spectra in SMD dyads are expected to contain only fundamentals and partial tones with no combination tones. Comparing SMD spectra with those recorded in SPD and TPD allowed us to highlight the possible presence of violin combination tones and their examination. Finally, the switching between players allowed us to draw some preliminary qualitative observations related to a given violin or player.

III. Experimental results and discussion

The results presented in this paper show that violin combination tones are present exclusively when the dyad is played by a single player whereas records with two players and summed notes show no signs of them at all. Violin combination tones occurred in all the following intervals analyzed (scientific pitch): perfect fifth ($C_5 - G_5$), perfect fourth ($C_5 - F_5$), major third ($C_5 - E_5$), minor third ($C_5^{\#} - E_5$), major sixth ($E_5 - C_6^{\#}$), minor sixth ($E_5 - C_6$), *tuono maggiore* ($G_5 - A_5$) and *tuono minore* ($A_5 - B_5$), all played in the key of C major except the last two, played in the key of G major. However, VCTs amplitude varied considerably among the various intervals and also between the two players. The analysis of this variability, which likely depends on several factors, was not pursued being not presently our main interest.

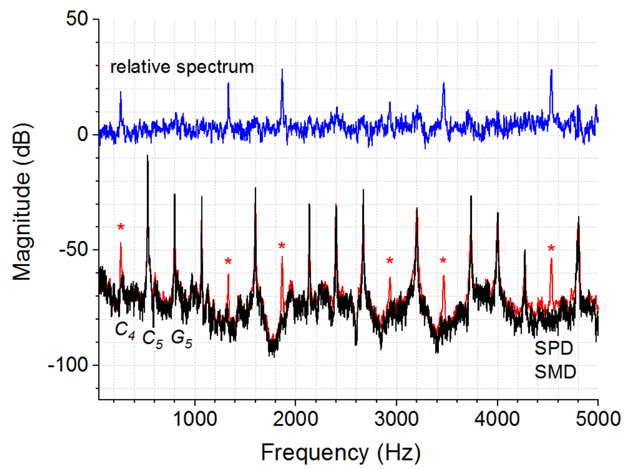

Figure 2 - Comparison of FFT spectra of SPD and SMD traces for the interval of perfect fifth $C_5 - G_5$. SPD and SMD are almost perfectly superimposed, except for the VCTs on the SPD trace (red line) identified by the asterisks. The upper trace shows the relative spectra $20 \cdot \log$(SPD/SMD) which highlights the difference between the SPD and SMD traces. C_4 corresponds to the VCT at the lower frequency.

Figure 1 shows the comparison between the FFT spectra of single player and summed note dyads for the interval of perfect fourth $C_5 - F_5$. The summed note spectrum (SMD) shows only peaks corresponding to the fundamentals and partial tones of the dyad, as expected, whereas the single player spectrum (SPD) shows several smaller peaks in between the fundamentals and partials. These peaks, marked with the asterisk, are the violin combination tones which are regularly distributed all along the spectrum.

If the frequencies f_1 and f_2 (with $f_1 < f_2$) are in a rational relation with $f_2 = f_1 \cdot m/n$ the pitch of all VCTs, expected from the distortion theory [3] is $hf_1 + kf_2 = (hn + km)f_1/n$, where h and k are integer numbers. Thus, the VCTs' peaks occur at frequencies that are integer multiples of f_1/n or the greatest common divisor (GCD) of f_1 and f_2 in accordance with [7].This can be seen in Figure 1 where VCTs repeat themselves every $f_2 - f_1$ interval corresponding to f_1/n ratio, named Fundamental Combination Tone (FCT), and equal in this case to 178Hz. Note that if f_1 and f_2 are not exactly in a rational relation, f_2 is equal to $f_1 \cdot m/n \pm e$, where the real number e is the small intonation error unintentionally caused by the players. The pitch of VCTs is then $hf_1 + kf_2 = (hn + km)f_1/n \pm ke$ and it does not coincide with GCD. In a few of the studied dyads the value of e was not negligible; however, this did not affect the presence of VCTs. The appearance of combination tones at regular and precise intervals shows that they occur as if they were the harmonics of a fundamental virtual note corresponding to the FCT plus or minus the amplified error.

Figure 2 shows the comparison between one player dyad and summed note dyad spectra, for the perfect fifth interval ($C_5 - G_5$). Traces are almost perfectly superimposed, except for the VCTs on the SPD trace (red line) identified by the asterisks, which emerge from the background. This aspect is even clearer in the relative spectrum trace ($20 \cdot \log$(SPD/SMD)) also shown in the figure, which highlights differences between the two traces by eliminating all the common features. Six VCTs peaks stand out clearly from the flat baseline.

The comparison between two players and summed note dyads spectra for the major sixth interval ($E_5 - C_6^{\#}$) is shown in Figure 3. It can be seen that TPD trace has no VCTs at all and it is very similar to SMD trace. The absence of VCTs is also clearly shown by the relative spectrum. This means that, as far as VCTs are concerned, the condition SPD (one player) and TPD (two plyers), both used by Tartini for investigating the third tone, are not equivalent.

The combination tones are present when a single player plays the dyad, but they disappear from the spectrum when the same dyad is played (one note each) by two violin players. Tartini stated that the third tone was heard equally well in both conditions, which means that combination tones generated by the violin were not relevant to his observations. Thus, the third tones he described were unaffected by VCTs being exclusively subjective combination tones due to cochlear distortion. A possible, although unlikely exception, is shown in Figure 2. The lowest pitched combination tones at C_4 is in Tartini's third tone range. Although we did not perform any proper listening experiments, for this particular dyad we noticed a clear difference between the SPD dyad played as it was recorded and after filtering out the narrow region of the VCT at C_4, which indicates that this VCT was perceivable.

Thus it may be asked why Tartini in his 1754 treatise did not mention this combination tone but rather indicated for the perfect fifth interval a third tone at unison with C_5. Tartini did not explain how the experient was performed, but if he used the configuration with two players, the third tone at C_4 was not mentioned simply because it was not there. Another possibility is connected with the players and violins he used since in our experience VCTs' amplitude depends on both these factors. Finally, it is interesting to note that after and even before the treatise of 1754, Tartini assigned the third tone not to C_5 but to C_4 [9] in perfect coincidence with the lowest pitched VCT shown in Figure 2. Thus, we cannot exclude that, at least for the interval of perfect fifth, the violin combination tone at C_4 might have influenced Tartini's listening. It is tempting to speculate that VCTs, with their variability, might have been at least partially responsible for the difficulties experienced by Tartini in attributing the right octave to some of the third tones perceived.

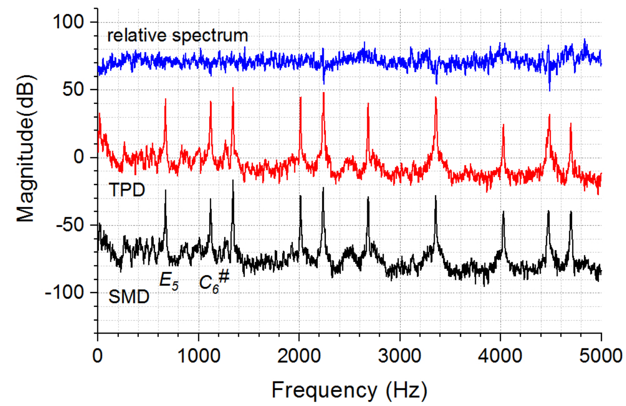

Figure 3 - Comparison of FFT spectra in configuration TPD (two violins, 2.7m apart) with SMD for the major six $E_5 - C_6^{\#}$. No VCTs peaks are present on the TPD trace which is almost perfectly superimposable to SMD. Small glitches in coincidence with some partial tones, are caused by differences in intonation and amplitude of the notes played in TPD and SMD. TPD and relative spectrum traces are shifted upward for clarity.

IV. Statistical Analysis

All the figures of the paper show clearly that VCTs exists only when the dyads are played by a single instrument and not when the same dyad is played by two instruments (one note each). To give more solidity to these observations we decided to investigate their statistical significance. This was done for those dyads which were played at least four times in all conditions. We report here the results for the perfect fourth ($C_5 - F_5$) and perfect fifth ($C_5 - G_5$) dyads. Spectra amplitude was measured on linear spectra (performed directly on the output of the sound card), in coincidence with the first six violin combination tones on TPD, SMD and TPD traces. An ANOVA test was then performed on the results. Means and SEM of the data obtained are shown graphically in Figure 4.

It is clear that spectra amplitude at VCTs is much greater in SPD than in TPD and SMD traces. Since the analysis showed that ANOVA *F* and *p* values were significant for all the VCTs, we performed a post-hoc Tukey test on the data. The results are summarized in Table 1. It can be seen that spectrum amplitude at VCTs of the dyad played by a single instrument (SPD), is significantly higher than that of the dyad played by two separated instruments (TPD) and of the summed note dyad (SMD). The data also show that SMD and SPD amplitudes are not significantly different from each other. Since SMD dyads have no VCTs, the same is true for TPD dyads. Similar results were also found for the VCTs from 7 to 12 of the perfect fourth interval of Figure 1, and for all the other intervals in which it was possible to measure VCTs (not shown here). These results confirm in a quantitative way the results showed by all the figures.

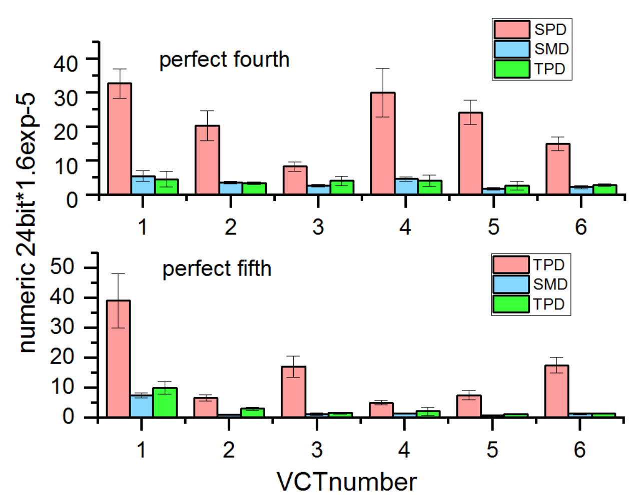

Figure 4 - Mean SEM (n = 4) of spectrum amplitude in correspondence of VCT$_1$(low frequency)-VCT$_6$ (high frequency) for the perfect fourth and perfect fifth intervals for condition SPD, SMD and TPD. Ordinate represents the numeric value of the 24 bit range multiplied by $1.6 \cdot 10^5$. Because of the VCT presence, peaks in SPD are is significantly greater than in TPD and SMD, whereas TPD and SMD are statistically not different.

V. Model and data simulation

Although the precise mechanism of combination tone generation by the violin is unknown, following Helmholtz, we know that any non-linearity in the sound production,

transmission or detection, produces a distortion that in principle can generate the combination tones. The summation of two sinusoidal signals at frequency f_1 and f_2 in a non-linear system leads to the production of additional frequencies including $2f_1$, $2f_2$, etc. (*harmonic distortion*) and $f_2 + f_1$, $f_2 - f_1$, etc. (*intermodulation distortion*).
In general, the frequencies generated are given by $hf_1 + kf_2$ (with h, k integer numbers different from zero). Based on this information, we investigated the response of a simple model in which the occurrence of combination tones was simulated by adding to the SMD record (containing no VCTs) the product of the two recorded waves (i.e. the signal of note 1 multiplied by the signal of note 2) scaled by a factor z, to introduce some intermodulation distortion in it. We called this SSPD. The results of the simulation with $z = 1/2$ for the perfect fifth interval ($C_5 - G_5$), are depicted in Figure 5.
It can be seen that the simulated traces reproduce rather well the experimental traces. Evaluation of the experimental and simulated relative spectra shows that their difference in any point of the graph is comparable to the noise, demonstrating the quality of this simple model. More complex intermodulation distortion models, including more harmonics and more orders, did not add more useful information to the analysis. The combination tones generated by the model with a periodicity of $f_2 - f_1$, encompasses fundamentals and harmonic tones in accordance with the theory which predicts the presence of VCTs in coincidence with the primary and partial tone peaks. This seems in contrast with the relative spectra in Figure 5 that shows no sign of coincident VCTs. This contrast however, is only apparent as the addition, for example, of the VCT at C_4 in Figure 5 to the fundamental at C_5 would produce an increment of less than 1dB, well below the noise floor.

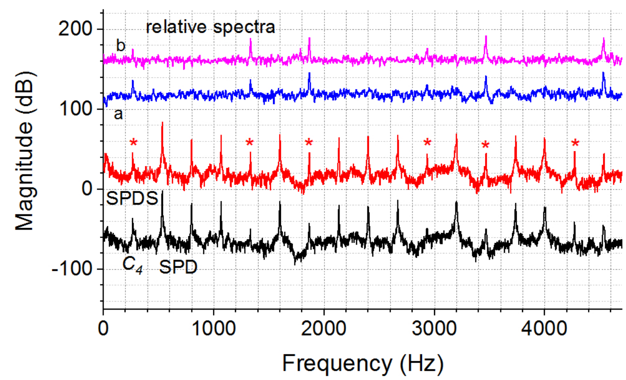

Figure 5 - FFT spectra for the interval of perfect fifth $C_5 - G_5$. Simulated Single Player Dyad (SSPD) represents the simulated response, traces a and b represent the experimental and the simulated relative spectra, $20 \cdot \log(\text{SPD}/\text{SMD})$ and $20 \cdot \log(\text{SSPD}/\text{SMD})$, respectively. Note the good similarity between experimental and simulated traces. Asterisks on SSPD trace, indicate VCTs peaks. Traces SPD, SSPD, originally superimposed to SPD trace, a and b both originally at zero dB, were shifted upwards for clarity.

The possible effects of the coincident VCTs cannot be determined when they are analyzed on the simple record SPD, as done by Lohri [7]. This is because coincident VCTs are superimposed on the peaks of fundamentals or partials whose amplitudes are unknown. Unlike that used by Lohri [7], our method of recording the SMD dyad in addition to SPD allows us, in principle, to measure VCTs easily. In fact, the subtraction between SPD and SMD records, as shown in Figure 5, allows us to isolate all the VCTs including the coincident ones, which can then be analyzed. It should be pointed out, however that to obtain the necessary precision, the amplitudes and frequencies of the notes in SPD and SMD, records, played necessarily at different times, needs to be the same within less than 1%. This precision cannot be obtained during normal violin playing but it can be obtained with a bowing machine. As a possible easier alternative the amplitude of the VCTs coincident with the partials could be calculated with the model simulation shown in Figure 5.

	Perfect fifth			**Perfect fourth**		
	SMD-SPD	TPD-SPD	TPD-SMD	SMD-SPD	TPD-SPD	TPD-SMD
VCT1	0.0018*	0.0025*	0.97	0.00071*	0.00008*	0.9
VCT2	0.00025*	0.0045*	0.26	0.0036*	0.0033*	0.98
VCT3	0.00098*	0.0012*	0.99	0.025*	0.034*	0.65
VCT4	0.00033*	0.0019*	0.38	0.017*	0.015*	0.99
VCT5	0.0011*	0.0021*	0.89	0.000016*	0.000063*	0.95
VCT6	0.00007*	0.000069*	0.99	0.00012*	0.00018*	0.94

Table 1 - p values computed with the post-hoc Tukey test for the comparison between spectrum amplitudes of the SPD, SMD and TPD traces in coincidence with combination tones VCT_1 (lower frequency) to VCT_6 (higher frequency) for the perfect fifth and perfect fourth intervals. Asterisks indicates statistical significance ($p < 0.05$).

Listening experiments were conducted by Lohri [7] to ascertain if a difference was perceived between records before and after the elimination (by a computer program) of the VCTs present. The results were somewhat unsatisfactory as the difference due to the VCTs presence was not perceived by all the listeners involved. The drawback of this method is that VCTs coincident with fundamentals and partials obviously cannot be eliminated so reducing the overall effect of VCTs. It seems reasonable to expect more convincing results by comparing SPD record, containing all the VCTs including the coincident ones, with TPD which contains no VCTs at all. Future experiments will be planned to verify this point.
As regarding the nature of the non-linearity inducing the intermodulation distortion, we can hypothesize many reasons for it, like bridge movement and violin top movement, to mention just two of them. Instead, the coupling of strings through the bow, seems to be excluded by the results of [7] and by our preliminary experiments (not reported here)

showing the presence of VCTs even when the dyad is played by stimulating the violin strings with an electromagnetic system similar to that described by Gough [10]. Clearly much more work is needed to identify the mechanism of the VCTs production.
Although not investigated in this paper, the variability of VCTs between different violins and players suggest the interesting possibility, which we have planned to investigate, that VCTs might contribute to violin sound quality.

VI. Conclusions

When a dyad is played on a violin, or when two notes are played separately on two violins, Combination Tones can be heard that are harmonics of the frequency difference of the two notes. This has already been reported by Tartini. Our research shows that when a dyad is played on a single violin, the Combination tones are also created by the instrument and can be found by FFT of the sound. When the dyad is played on separate instruments, the Combination Tones can also be heard, as they are created in our cochlea, but are not recorded by the microphone and do not show up in the FFT. Our work underwrites the observations of Tartini, but enriches it by demonstrating that the Combination Tones, as heard by our ears, are due in part to the non-linearity (creation of Combination Tones) of our ears as well as to the non-linearity of the instrument. The observation that Tartini did not find any difference between these two conditions, suggests that violin combination tones in his experiments were too small to be perceived. Planned future work will be made to investigate to what extent violin combination tones correlate with subjective sound quality.

Acknowledgments

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Bibliography

1. G. Tartini, *Trattato di musica secondo la vera scienza dell'armonia*. Stamperia del seminario, Padova, 1754.
2. G. Vieth, "Ueber combinationstöne, in beziehung auf einige streitschriften über sie zweier englischer physiker, th. young und jo. gough," *Annalen der Physik*, vol. 21, no. 11, pp. 265–314, 1805.
3. H. v. Helmholtz, *On the Sensations of Tone as a Physiological Basis for the Theory of Music*. Longmans, Green, and Co., 1877, translated in English by Ellis, Alexander J.
4. R. Plomp, "Detectability threshold for combination tones," *Journal of the Acoustical Society of America*, vol. 37, no. 6, pp. 1110–1123, 1965.
5. J. L. Goldstein, "Auditory nonlinearity," *Journal of the Acoustical Society of America*, vol. 41, no. 6, p. 676, 1967.
6. L. Robles and M. A. Ruggero, "Mechanics of the mammalian cochlea," *Physiological Reviews*, vol. 81, no. 3, pp. 1305–1352, 2001.
7. A. Lohri, S. Carral, and V. Chatziioannou, "Combination tones in violins," *Archives of Acoustics*, vol. 36, no. 4, pp. 727–740, 2010.
8. A. Lohri, *Kombinationstöne und Tartinis "terzo suono"*. Schott Music GmbH et Co. KG, 10 2016.
9. P. Barbieri, "Tartinis dritter ton und eulers harmonische exponenten. mit einen unveröffentlichen manuscript tartinis," *Musiktheorie*, vol. 7, no. 3, pp. 219–234, 1992.
10. E. C. Gough, "The resonant response of a violin g-string and the excitation of the wolf-note," *Acta Acustica united with Acustica*, vol. 44, no. 2, pp. 113–123, 1980.